Estimating Productivity in the Presence of Spillovers: Firm-level Evidence from the US Production Network*

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Abstract

This paper examines the extent to which efficiency gains diffuse over a network of firms. Empirical studies typically estimate firm-to-firm spillovers using productivity measured from estimated production functions. However, standard production function estimation methods implicitly rule out the interdependence of firms' outcomes and decisions through productivity spillovers. I show that

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ignoring network effects when estimating production functions biases productivity and spillovers in directions that cannot generally be predicted *a priori*. To address this limitation, I develop a framework to jointly estimate network effects and firm-level productivity. My approach accounts for non-random buyer-supplier matching and network effects that vary by relationship direction and firm characteristics. Using this method, I characterize productivity spillovers over the US production network from 1977 to 2016 and find substantial heterogeneity by direction, industry, firm size, and over time. My results suggest that the average firm in 1978 would be 16% more productive by 2016 due to spillovers. In addition, a 10% increase in the productivity of the most central firm in each year could result in up to a 2% rise in aggregate total factor productivity through spillovers alone. These findings highlight the potential for industrial policy, even if narrowly targeted, to have broader ramifications for the economy as a whole.

1 Introduction

Production function estimation is at the heart of several important questions in economics. From examining changes to market power and assessing the impact of trade liberalization, to decomposing the sources of aggregate productivity growth, understanding firms' decisions and their implications on market outcomes often hinges on the accurate measurement of total factor productivity (TFP).[1]

A significant finding of the literature on firm-level productivity is that businesses exhibit marked differences in TFP, even within narrowly-defined industries, and a vast body of work has sought to explain this dispersion.[2] One possible explanation is that firms may affect one another in ways that do not show up in the prices of intermediate goods and services; they may experience spillovers from knowledge transfers or agglomeration externalities. For example, in the trade literature, firms have been found to impact the productivity of counterparts through activities such as foreign direct investment (FDI) and exporting.[3] Javorcik (2004) found that FDI in Lithuania had a positive effect on the productivity of domestic firms through backward linkages, and Keller and Yeaple (2009) documented the existence of horizontal spillovers from multinationals to US firms. Likewise, Alvarez and López (2008) provided evidence from Chile of positive productivity spillovers from domestic and foreign-owned exporters on their suppliers, Alfaro-Urena et al. (2022) found TFP gains of $4\% - 9\%$ among Costa Rican firms after they began to supply to multinational corporations, and Amiti et al. (2023) documented significant productivity increases for Belgian firms supplying to superstar firms including multinationals, exporters and large firms.

The present paper quantifies the transmission of productivity gains through buyer-supplier relationships in the United States and examines how the existence of spillovers affects the measurement of TFP. I consider spillovers not just from firm activities, but directly from productivity as well. A firm's TFP could increase or decline due to the productivity of the firms with which it has a relationship. The expected direction of this effect is not immediately clear: firms may learn from their peers and become more productive or might free-ride on their trading partners' efficiency. Empirical investigations into direct efficiency spillovers are relatively new. Serpa and Krishnan

[1] See De Loecker and Syverson (2021) for a primer on productivity analysis.
[2] See Syverson (2011) for a review.
[3] See Keller (2010) for a review of the evidence on spillovers from FDI and exporting.

(2018) examined this question with data on firm-level buyer-supplier relationships in the US, whereas Bazzi et al. (2017) used input-output matrices to construct measures of the relationships between Indonesian firms. Both studies found that firms enjoy significant boosts to productivity from their relationships with more productive counterparts.

However, an important gap exists in the literature on productivity spillovers. Many studies assess the existence of spillovers using TFP estimates obtained from semi-parametric proxy variable/control function approaches. Introduced by Olley and Pakes (1996) and refined in Levinsohn and Petrin (2003), Wooldridge (2009), and Ackerberg et al. (2015) (hereafter OP, LP, Wooldridge and ACF respectively), these methods assume that a firm's future productivity depends only on its own past productivity and characteristics. Alternative methods such as Gandhi et al. (2020) that relies on first order conditions for identification also make the same assumption on the productivity evolution process. This implies that each firm's productivity evolves independently, and implicitly rules out the existence of anticipated spillovers.

The contributions of this paper are three-fold. First, I show that when productivity spillovers exist, failing to account for this interdependence could lead to biased estimates of production function elasticities and TFP. Using Monte Carlo experiments, I demonstrate that input elasticities are generally not consistent when the law of motion for productivity precludes spillovers. As De Loecker (2013), De Loecker et al. (2016), and Garcia-Marin and Voigtländer (2019) point out, our conclusions about what drives changes in productivity are sensitive to how it is measured. De Loecker (2013) showed that measuring TFP under standard assumptions can lead us to underestimate the impact of exporting on productivity. In Garcia-Marin and Voigtländer (2019), the downward bias in learning-by-exporting estimates arises from revenue-based productivity measures that cannot disentangle the lower prices firms charge upon entry into export markets from their increased efficiency. Unfortunately, the direction of bias in spillover estimates is not as clear-cut. I find that, depending on the structure of the network and persistence of productivity over time, estimating spillovers on mismeasured TFP can lead us to *overestimate* network effects in some cases and *underestimate* them in others.

Second, I propose a modification to standard control function and first order condition approaches that flexibly accounts for the presence of spillovers. To do so, I apply results from the peer effects and spatial econometrics literature including Lee

(2003); Bramoullé et al. (2009); Lee and Yu (2016), with an important distinction: these papers deal with observed outcomes, whereas I jointly estimate the outcome and spillovers. This comes at the cost of a few additional assumptions that are, nonetheless, compatible with both the standard production function and network effects frameworks. A few studies have explicitly allowed for cross-sectional dependence between firms through linkages in TFP estimation. Malikov and Zhao (2021) specified a framework in which a firm's productivity depends on the lagged productivity of nearby firms within the same industry to study inward FDI effects in China's electric machinery manufacturing sector. Merlevede and Theodorakopoulos (2023) examined the impact of intangibles between European firms in ownership networks by allowing affiliates to be influenced by the productivity of their parent companies and vice versa.

My proposed approach offers three advantages relative to the existing literature. First, I allow network effects to occur within the same period and do not impose that spillovers only occur with a lag. Contemporaneous spillovers have been found to matter empirically[4] and this approach can plausibly capture these effects while also accommodating additional lagged effects. Second, I account for common shocks that are localized within the network, which typically confounds the estimation and interpretation of spillovers. Third, my approach can accommodate network formation that is endogenous to productivity shocks. The estimator is therefore useful in a broad range of contexts without overly strong assumptions. I show through Monte Carlo simulations that the estimator does not generate spurious network effects and provides consistent estimates of production functiu elasticities. I extend the framework to examine heterogeneous spillovers in the manner of Dieye and Fortin (2017) and Patacchini et al. (2017), that vary by the nature of the relationship between firms and their characteristics.

Third, I apply this methodology to examine the transmission of efficiency gains through the production network of publicly listed firms in the United States from 1977 to 2016. I find evidence of positive productivity spillovers, with a stronger impact from suppliers to customers, and substantial heterogeneity by sector and firm size. Estimates suggest that if the most connected firm in a given year was 10% more productive, spillovers would lead to an increase in aggregate TFP of 0.2% to

[4]For example, (Keller, 2010) found that horizontal productivity spillovers from inward FDI in the US firms showed up within the same period and with a one-year lag.

1.9%. Furthermore, the cumulative impact of spillovers over time implies that the average firm in 1978 would be 16% percent more productive by 2016 due to spillovers alone. Decomposing the spillovers by sector, shows that electronics manufacturers have benefited from almost all other sectors, whereas retailers are important sources of efficiency gains to trading partners in other sectors.

My results highlight an additional channel for industrial policy to affect economic growth. Given that a substantial portion of these spillovers can be attributed to distribution and information technology, policymakers could target high-growth sectors that can generate these second-order effects. Furthermore, the centrality of a few firms to the production network suggests that policies adversely impacting such firms could have broader negative ramifications for the US economy.

In the next section, I describe the data and features of the sample of the US production network that I observe. Section 3 presents my empirical framework and discusses the biases that arise from ignoring spillovers in the standard control function approach. In section 4, I propose a procedure for estimating production functions in the presence of various network effects and clarify the assumptions needed to obtain valid estimates. I introduce a model of network formation in section 5 to account for endogenous network selection. Section 6 demonstrates the advantages of my approach over existing methods using Monte Carlo experiments. I consider extensions to the benchmark model including a gross output production function in section 7. Section 8 presents my empirical results and section 9 concludes.

2 Data: The US Production Network

I begin by describing the data with which I characterize the firm-level production network within the US, to highlight features that are important for my empirical methodology. To examine the magnitude and origins of productivity spillovers in the US, I rely on a panel of publicly-listed firms in the *Compustat* database from 1977 to 2016. *Compustat* collects companies' financial statements from form 10-K reports filed with the US Securities and Exchange Commission (SEC). This provides detailed information on firms' sales, capital stock, expenses and employees. I supplement this with industry-level deflators and wages from the US Bureau of Economic Analysis (BEA) to construct the necessary variables to estimate a production function.[5]

[5]See section E in the appendix for further details on variable construction.

Information on buyer-supplier links also comes from 10-K reports. Statement no. 14 issued in December 1976 by the Financial Accounting Standards Board (FASB) requires each firm to report any customers that are responsible for 10% or more of its sales within a fiscal year. I conservatively match the reported customer names to company financial data. The resulting network contains 18,872 unique buyer-supplier pairs and 66,052 dyad-year observations.[6]

I restrict the firm-level sample to the businesses that either report or are reported as customers, and have positive values of sales, capital stock, labor, and materials. I discard firms in agriculture, forestry and fishing, because these industries have too few observations in both the firm- and dyad-level datasets. This yields an unbalanced panel of 8,353 firms and 55,047 firm-year observations.

Table 1 reports average firm characteristics by decade and over the full sample. Due to the nature of the firms in question, and the restriction to companies with customer or supplier data, firms in the sample tend to be large, averaging 19,000 employees and $6.08 billion in annual sales. Based on the BEA's classification of large enterprises as firms employing 500 or more workers, about two-thirds of the sample are large firms. As shown in table 2, manufacturers comprise more than half of the firms in the sample. Information and Services are the next largest sectors represented in the sample.[7]

The observed sample of the production network is sparse; that is, the number of connections per firm is low. Figure 1 shows that firms report 1 or 2 customers on average, whereas the same customers are reported by about 3 or 4 suppliers. Consistent with the 10% sales reporting requirement, reported customers tend to be large; the average customer realizes about eight times as much in sales as the average supplier in the data (see figure 5). This may be due to two factors: relatively small firms are likely to have major customers and larger firms are likely to be major customers. However, although the value traded in the average reported relationship is sizable and increases over time, figure 2 indicates each individual relationship makes up a declining share of the suppliers' sales.

In figure 4, I examine features of the network that affect the identification of spillovers within my framework. Network density, measured by the number of ob-

[6]Other studies that have used this dataset to study the US production network include Atalay et al. (2011), Lim et al. (2017) and Serpa and Krishnan (2018). I am grateful to the authors of Atalay et al. (2011) for graciously sharing their matched buyer-supplier data with me.

[7]See section E for a full list of industries in each sector.

Table 1: Firm Characteristics

	1977-1986	1987-1996	1997-2006	2007-2016	Full Sample
Sales	3.29	3.65	6.45	10.57	6.08
	(12.27)	(14.64)	(22.17)	(28.9)	(21.03)
Sales per 1000 employees	0.29	0.34	0.51	0.83	0.5
	(1.16)	(0.82)	(3.63)	(7.19)	(4.12)
Value Added	0.93	1.02	1.88	3.47	1.85
	(2.9)	(3.04)	(5.34)	(8.23)	(5.47)
Capital stock	3.23	3.52	4.89	9.79	5.4
	(12.01)	(14.49)	(18.13)	(33.22)	(21.41)
Materials	2.53	2.85	4.73	7.12	4.37
	(11.06)	(13.45)	(18.86)	(22.98)	(17.56)
Employees (thousands)	15.35	13.9	19.15	27.22	18.95
	(48.73)	(43.41)	(61.46)	(84.71)	(62.02)
Large firm (employees $\geq$ 500)	0.65	0.63	0.68	0.77	0.68
Observations	10339	15268	15495	13455	54557

Notes: This table reports average characteristics of firms in the sample. Standard deviations are in parentheses. All monetary values are in 2009 billion USD.

Table 2: Industry Composition

	1977-1986	1987-1996	1997-2006	2007-2016	Full Sample
Mining	9.2	5.8	4.0	7.4	6.3
Utilities	6.9	5.3	2.8	3.4	4.4
Construction	0.9	0.8	0.9	1.0	0.9
Durables Manufacturing	25.4	21.4	19.1	16.8	20.4
Non-Durables Manufacturing	17.6	17.7	18.6	19.7	18.4
Electronics Manufacturing	14.9	17.9	19.7	16.8	17.6
Wholesale	3.3	4.7	4.3	3.7	4.1
Retail	3.6	4.3	4.6	4.7	4.4
Transport and Warehousing	4.4	3.3	3.4	4.3	3.8
Information	5.5	8.9	10.5	10.2	9.0
Finance, Insurance & Real Estate	2.7	2.8	3.3	4.9	3.5
Services	5.6	7.1	8.9	6.9	7.3
Total	100	100	100	100	100

This table reports the distribution of firms in the sample by primary sector as determined by the BEA industry classification.

served links as a fraction of all possible links, does not exceed 0.28% in any year. The network gets sparser at the beginning of the sample and denser after the mid-90s. At the same time, network transitivity, the number of observed triads as a share of all possible triads, trends upward throughout the sample, but does not exceed 1.2%. In sections 3 and 4, I discuss the importance of density and transitivity for both the biases in input elasticities from standard approaches and the performance of my proposed estimator.

Each year, the production network is often dominated by a large cluster of firms connected to one another. Figure 3 shows that the number of edges in the largest connected component as a share of all edges in the network ranges from 56% to 70%. This is largely due to the presence of a few well-connected firms, whereas the remainder of the network consists of peripheral, small clusters.

Variations in clustering patterns over time reflect changes in the relative importance of each industry. Figure 7 reports the ten most central firms as measured by the number of links a firm has as a share of all observed links. In the first ten years of the sample, manufacturers of automotives and other durable goods dominated the list. In the next decade, AT&T rose to the top of the list, and electronics manufacturers such as IBM had begun to emerge. In the 1997-2006 period, Walmart rose to the top the list, and while automotive and electronics manufacturers still featured at the top of the centrality distribution, their centrality had declined relative to earlier decades. By the end of the sample, retailers and wholesalers had superseded most manufacturers, with Walmart continuing to top the list.

Figure 6 shows the relationship between a firm's labor productivity, as measured by the natural log of sales per employee and that of its average buyer or seller. The slope of the fitted regression line is 0.38, indicating a strong positive correlation between the two quantities. Interpreting this relationship requires distinguishing among several possible explanations. Foremost is the question of direction: does a firm become more efficient by learning from its neighbors, or does causation move in the opposite direction? And if a firm is simultaneously affected by and affecting its partners, how can one pin down the magnitude of the effect? On the other hand, this relationship may be driven by the sorting of firms; if more productive firms trade with one another, then this correlation is evidence of network formation rather than spillovers. Yet another possibility is that supply chains are a transmission channel for production and demand shocks, inducing the revenues of connected firms to move in

the same direction.

Each of these explanations has different implications for how productivity is measured: if there are spillovers due to learning, then firms' input decisions will likely be influenced by the efficiency of their suppliers or buyers, whereas unanticipated common shocks are unlikely to affect input choices to the same degree. In the next section, I introduce an empirical framework with the goal of distinguishing among these channels, examining how they impact the measurement of TFP, and quantifying the direction and magnitude of productivity spillovers.

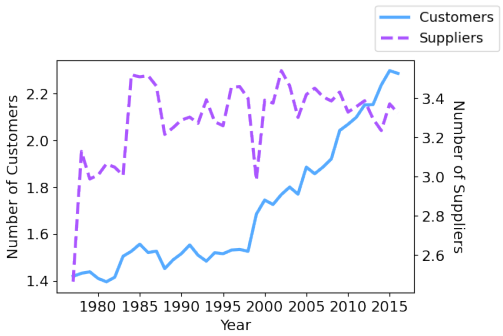


Figure 1: Average Firm Degree

This figure shows annual average out- and in-degrees (customers and suppliers) for firms in the sample.

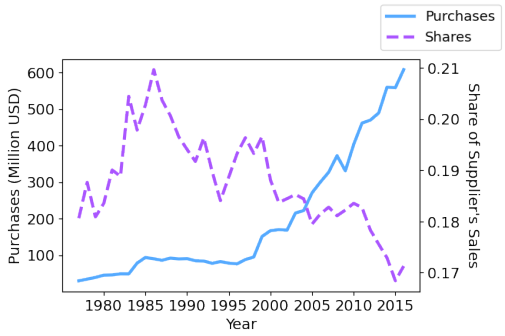


Figure 2: Value Traded in Relationships

This figure shows the annual average value traded by each buyer-supplier pair in nominal Million USD and as a share of each seller's total sales.

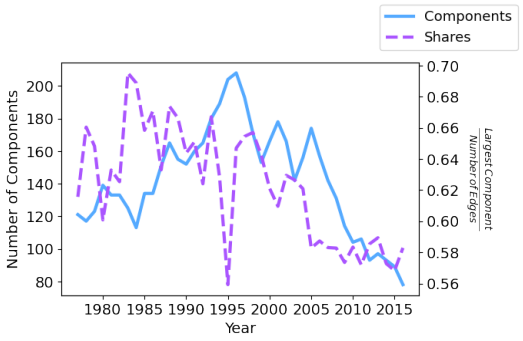


Figure 3: Network Clustering and Components

This figure shows the number of connected components and the largest component as a share of all edges in the network sample over time.

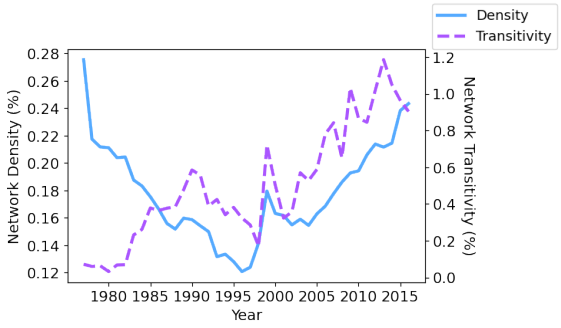


Figure 4: Network Density and Transitivity

This figure shows the density and transitivity of the network sample over time.

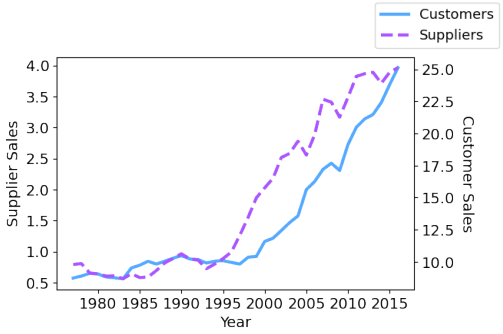


Figure 5: Customer and Supplier Sales

This figure shows annual average sales (in 2009 Billion USD) of firms reporting and reported as customers in the sample.

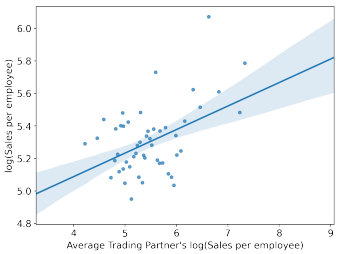


Figure 6: Relationship between Labor Productivity of Firms and their Trading Partners

This figure is a binned scatter plot showing the relationship between the labor productivity of a firm and its average trading partner. The slope of the fitted regression line is 0.38.

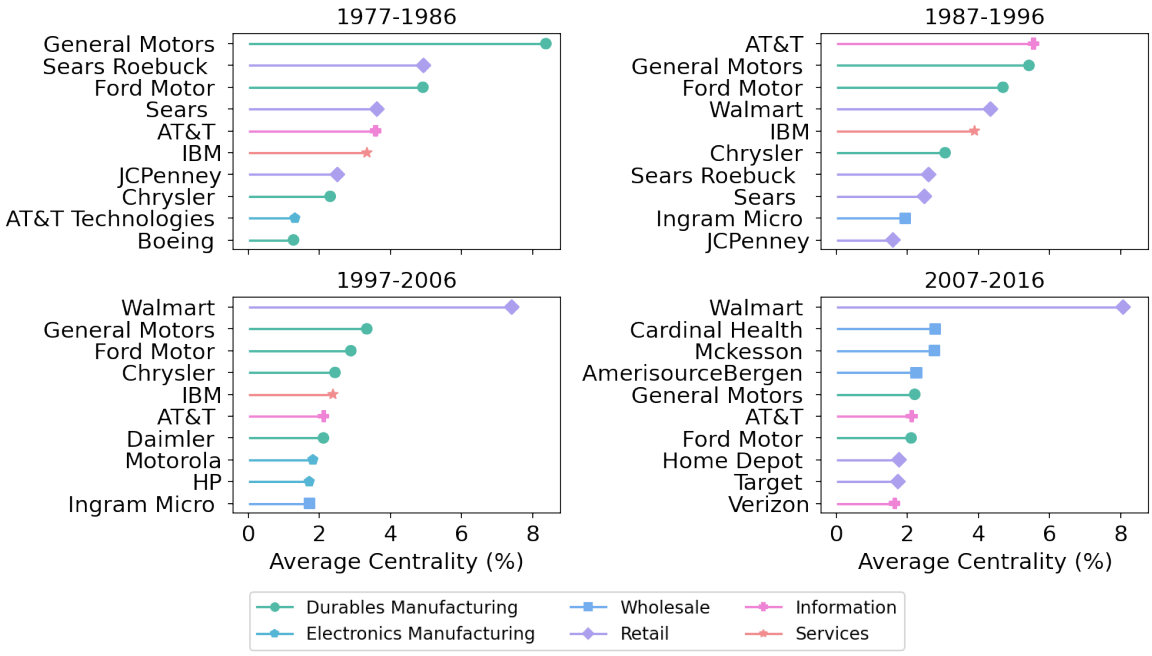


Figure 7: Firm Centrality
This figure shows the top ten firms by average centrality in each period.

3 Empirical Framework

Consider a production technology for firm i in period t in which productivity is Hicks-neutral:

$$Y_{it} = F(L_{it}, K_{it})e^{\omega_{it}+\varepsilon_{it}} \tag{1}$$

where output, Y_{it} is a function of labor, L_{it} and capital, K_{it}. Output is shifted by an exogenous shock, $e^{\varepsilon_{it}}$ independent of all variables known to the firm by the end of the period, the information set, $\mathcal{I}_{it}$. $e^{\omega_{it}}$ is firm-specific TFP that is unobserved by researchers but known to the firm when making production decisions. $F(\cdot)$ is known up to some parameters. Taking the natural log of (1) yields:

$$y_{it} = f(l_{it}, k_{it}) + \omega_{it} + \varepsilon_{it} \tag{2}$$

The main limitation to estimating $f(\cdot)$ is a simultaneity problem: firms choose their inputs based on the realization of ω_{it}. Therefore, simply regressing a firm's

output on its inputs would lead to a biased estimate of $f(\cdot)$.

To address this issue, the control function/proxy variable approach makes a set of assumptions on timing, a proxy variable and how productivity evolves over time. The existence of spillovers primarily poses a problem for the last set of assumptions. Productivity is typically assumed to follow a first-order Markov process:

$$\omega_{it} = h(\omega_{it-1}) + \eta_{it} \tag{3}$$

where $h(\cdot)$ is unknown and η_{it} is mean independent of firm's information set at the beginning of the period $\mathcal{I}_{it-1}$. Suppose instead that ω_{it} is affected by some other firm j either through its past decisions $\boldsymbol{x_{jt-1}}$ and/or its current productivity, ω_{jt}:

$$\omega_{it} = h(\omega_{it-1}, \boldsymbol{x_{jt-1}}, \omega_{jt}) + \zeta_{it} \tag{4}$$

where $E[\zeta_{it}|\mathcal{I}_{it-1}] = 0$. The effect of $\boldsymbol{x_{jt-1}}$ represents spillovers from firm $j's$ activities such as research and development (R&D), FDI, exporting, etc. The inclusion of ω_{jt} indicates that j being more productive could contemporaneously influence i's productivity, and both firms' TFP may be jointly realized in the same period. Since firm j's TFP is also determined by its past productivity, ω_{jt-1}, this representation allows for productivity spillovers to have dynamic implications, while allowing for the possibility that firm i is also affected by random shocks, ζ_{jt} to j's productivity within the same period.

When researchers estimate TFP under the assumption in equation (3) whereas the true process is represented by equation (4), then the effect of firm j on i is attributed to η_{it}, which now violates the conditional independence assumption. In the following subsections, I examine the biases arising from standard control function approaches in greater detail.

Accounting for $\boldsymbol{x_{jt-1}}$ is fairly straightforward if we assume that it is known to i at the beginning of the period; that is, $\boldsymbol{x_{jt-1}} \in \mathcal{I}_{it-1}$. However, ω_{jt} poses a more serious challenge because it is jointly realized with ω_{it} and cannot therefore be assumed to be in $\mathcal{I}_{it-1}$. In section 4, I outline the assumptions needed to properly account for the effect of ω_{jt} on ω_{it} when estimating production functions.

3.1 Control Function Approach

Suppose $f(\cdot)$ takes the form of a simple structural value-added Cobb-Douglas production function as in Ackerberg et al. (2015):[8]

$$y_{it} = \alpha_\ell \ell_{it} + \alpha_k k_{it} + \omega_{it} + \varepsilon_{it} \tag{5}$$

where y_{it}, k_{it}, and ℓ_{it} are the logs of value-added[9], capital, and labor respectively. Obtaining consistent estimates of $\boldsymbol{\alpha}$ and ω_{it} requires three sets of assumptions.

The first relates to the timing of firms' decisions. Capital is a state variable, determined in the preceding period as a deterministic function of the firm's previous capital stock and its investment decision: $k_{it} = \kappa(k_{it-1}, i_{it-1})$. Labor, on the other hand, may or may not have dynamic implications. It may be fully adjustable and chosen after productivity is realized, or partly (or wholly) determined in the previous period. However, it needs to be chosen prior to the intermediate input decision. Based on its current capital stock, workforce, and productivity, the firm chooses intermediate inputs according to the following function:

$$m_{it} = \mathbb{M}(k_{it}, \ell_{it}, \omega_{it})$$

Next, one needs to assume that the demand for materials, $g(\cdot)$ is strictly monotonic in productivity, and that productivity is the only unobservable component of the input demand function. This guarantees that TFP can be expressed solely as a function of observables $\omega_{it} = \mathbb{M}^{-1}(k_{it}, \ell_{it}, m_{it})$. Substituting into the production function yields the following:

$$y_{it} = \alpha_\ell \ell_{it} + \alpha_k k_{it} + \mathbb{M}^{-1}(k_{it}, \ell_{it}, m_{it}) + \varepsilon_{it} \tag{6}$$

Although α_k and α_ℓ are not identified in this equation, we can obtain consistent

[8] I choose ACF because it allows for relatively flexible assumptions on the data-generating process for output, capital, labor and materials. However, this critique applies more broadly to OP, LP, Wooldridge and first order condition approaches such as Gandhi et al. (2020) that rely on similar assumptions on the productivity evolution process.

[9] Output minus intermediate inputs.

estimates of the firm's expected value-added:

$$E[y_{it}|\mathcal{I}_{it}] = \varphi_{it} = \alpha_\ell \ell_{it} + \alpha_k k_{it} + \omega_{it} \tag{7}$$

This disentangles productivity from the idiosyncratic shock ε_{it}. To identify capital and labor elasticities, the evolution process for productivity must be specified. A standard assumption is that productivity follows a first-order Markov process given its information set $\mathcal{I}_{it-1}$ in the previous period:

$$\omega_{it} = h(\omega_{it-1}) + \eta_{it} \tag{8}$$

where $E[\omega_{it}|\mathcal{I}_{it-1}] = E[\omega_{it}|\omega_{it-1}] = h(\omega_{it-1})$. $h(\cdot)$ is known to the firm but unobserved by the researcher, whereas η_{it} is idiosyncratic. Given equation (7), I can write lagged productivity as:

$$\begin{aligned} \omega_{it-1} &= \varphi_{it-1} - \alpha_k k_{it-1} - \alpha_\ell \ell_{it-1} \\ \implies \omega_{it} &= h(\varphi_{it-1} - \alpha_k k_{it-1} - \alpha_\ell \ell_{it-1}) + \eta_{it} \end{aligned}$$

Substituting into the production function yields:

$$y_{it} = \alpha_\ell \ell_{it} + \alpha_k k_{it} + h(\varphi_{it-1} - \alpha_k k_{it-1} - \alpha_\ell \ell_{it-1}) + \eta_{it} + \varepsilon_{it}$$

Since $E[\varepsilon_{it}|\mathcal{I}_{it}] = 0$ and $E[\eta_{it}|\mathcal{I}_{it-1}] = 0$ by assumption, then we can identify α_k, α_ℓ based on the moment restriction:

$$E[\varepsilon_{it} + \eta_{it}|\mathcal{I}_{it-1}] = E[y_{it} - \alpha_k k_{it} - \alpha_\ell \ell_{it} - h(\varphi_{it-1} - \alpha_k k_{it-1} - \alpha_\ell \ell_{it-1})|\mathcal{I}_{it-1}] = 0 \tag{9}$$

Using this equation, we can derive moment conditions to estimate the elasticities. Because there are three unknowns, $(\alpha_k, \alpha_\ell, h(\cdot))$, a typical set of moments would be:

$$E[(\eta_{it} + \varepsilon_{it})k_{it}, \ell_{it-1}, \varphi_{it-1}] = 0 \tag{10}$$

3.2 Network Effects

To examine biases due to the existence of spillovers, we need to first understand how network effects are characterized. Within a given year, relationships between n_t firms result in a network. This can be represented by an $n_t \times n_t$ adjacency matrix A_t such that $A_{ij,t} = 1$ if firm i has a relationship with firm j in that year and zero otherwise. These relationships could be transactional (i sells inputs to j) or some other form of firm interdependence, such as i and j sharing a board member. The adjacency matrix need not be symmetric. As is standard in the peer-effects literature, I impose $A_{ii,t} = 0$ for all i so that a firm cannot have a spillover effect on itself.

In most examples, I focus on buyer-supplier networks, but this framework could apply to other types of inter-firm relationships.[10] Suppose we are interested in how upstream firms are affected by the productivity of their downstream network. Let N_{it} be the set of i's customers in period t and $n_{it} = |N_{it}|$.[11] We would like to estimate the following network effects equation:

$$\omega_{it} = \beta_1 + \rho\omega_{it-1} + \boldsymbol{x_{it-1}\beta_x} + \lambda\frac{1}{n_{it}}\sum_{j\in N_{it}}\omega_{jt} + \frac{1}{n_{it}}\sum_{j\in N_{it}}\boldsymbol{x_{jt-1}\beta_{\bar{x}}} + c_{\psi_t} + \zeta_{it} \qquad (11)$$

where x_{it-1} is a $1 \times k$ vector of exogenous firm characteristics that could influence productivity, such as past R&D or exporting.

In this equation, there are three ways in which firm i's network could be related to its productivity. In the terminology of Manski (1993), the first channel is *endogenous network effects*: a firm's productivity is affected by the average productivity of its neighbors. This is measured by λ.

The second mechanism is *contextual effects* captured by $\boldsymbol{\beta_{\bar{x}}}$. Firms may be influenced by the characteristics or activities of their neighbors. For example, a firm's R&D could generate positive productivity spillovers on its business partners.

A firm's relationships could also result in *correlated effects*, productivity shocks common to all firms in a network cluster. Let ψ_t index the sub-components of a network in period t, that is firms who are at least indirectly connected to each other. Then c_{ψ_t} is a correlated effect for all firms in component ψ_t.

[10] Provided the network satisfies certain conditions for identification. See the rest of this section for details.

[11] Note that for some final goods producers and retailers, $n_{it} = 0$. These firms may not experience spillovers from others, but could still affect their suppliers.

An underlying assumption here is that the network is exogenous; that is, firms do not select partners in ways that are systematically correlated with their productivity. For now, I abstract from network selection and address it in section 5.

For the rest of this discussion, it would be convenient to rewrite these equations in matrix notation. Define the interaction matrix G_t as the row-normalized form of A_t.[12] Equation (11) can be rewritten as:

$$\omega_t = \beta_1 \iota + \rho \omega_{t-1} + \boldsymbol{x_{t-1}} \boldsymbol{\beta_x} + \lambda G_t \omega_t + G_t \boldsymbol{x_{t-1}} \boldsymbol{\beta_{\bar{x}}} + c_{\psi_t} + \zeta_t \tag{12}$$

The reduced form is as follows:

$$\omega_t = (1 - \lambda G_t)^{-1} (\beta_1 \iota + \rho \omega_{t-1} + \boldsymbol{x_{t-1}} \boldsymbol{\beta_x} + G_t \boldsymbol{x_{t-1}} \boldsymbol{\beta_{\bar{x}}} + c_{\psi_t} + \zeta_t) \tag{13}$$

$|\lambda| < 1$ implies that we can represent $(I - \lambda G_t)^{-1}$ as a geometric series.

$$\omega_t = \sum_{s=0}^{\infty} \lambda^s G_t^s (\beta_1 \iota + \rho \omega_{t-1} + \boldsymbol{x_{t-1}} \boldsymbol{\beta_x} + G_t \boldsymbol{x_{t-1}} \boldsymbol{\beta_{\bar{x}}} + c_{\psi_t} + \zeta_t) \tag{14}$$

Bramoullé et al. (2009) prove that (12) is identified if the identity matrix I, G and G^2 are linearly independent. The presence of intransitive triads[13] guarantees that linear independence holds. Production networks naturally have this structure because supply-chains tend to be unidirectional. Therefore, if ω_t was observed, one could estimate (12) using 2SLS (Lee, 2003; Bramoullé et al., 2009), QMLE (Lee and Yu, 2016) or Bayesian methods in (Goldsmith-Pinkham and Imbens, 2013).

Measuring productivity adds a layer of complexity to the problem. A typical strategy as in Javorcik (2004) and Serpa and Krishnan (2018), is to first obtain TFP values by estimating a production function such as using a method described above, and use these estimates in the network effects equation in (12). However, these approaches implicitly rule out the presence of spillovers, and the resulting TFP estimates are incompatible with the a wide set of network models nested in the peer effects model above.

[12] $G_{ij,t} = 1/n_{it}$ if $A_{ij,t} = 1$ and zero otherwise.

[13] An intransitive triad in a graph is a set of nodes i, j, k, such that i is connected to j and j to k, but k is not connected to i.

3.3 Biases due to Network Effects

When productivity is affected by network effects, the independence assumption on the productivity shock is violated. However, the impact on the estimation of production function elasticities will differ by the type of effect.

Suppose TFP is estimated under the exogeneity assumption in (3) but the true process is given by equation (12). This implies:[14]

$$E[\eta_t|\mathcal{I}_{t-1}] = \boldsymbol{x_{t-1}}\boldsymbol{\beta_x} + \lambda G_t E[\omega_t \mid \mathcal{I}_{t-1}] + G_t\boldsymbol{x_{t-1}}\boldsymbol{\beta_{\bar{x}}} + E[c_{\psi_t} \mid \mathcal{I}_{t-1}]$$

In general, this expression is not equal to zero. $\boldsymbol{x_{t-1}}$ is a source of omitted variable bias but De Loecker (2013) and Gandhi et al. (2020) show that the productivity process can be modified to account for its impact, as long as $\boldsymbol{x_{t-1}}$ is in the firm's information set at the beginning of the period.[15] Contextual effects can be accounted for in the same way under similar assumptions. Provided that network formation is exogenous, including $G_t\boldsymbol{x_{t-1}}$ in equation (3) would eliminate bias from this dimension.

$G_t E[\omega_t|\mathcal{I}_{t-1}]$ poses a serious challenge because in general, $E[\omega_t|\mathcal{I}_{t-1}] \neq 0$. Consider the correlation between neighbors' current productivity and current capital stock. Using the reduced form of $G_t\omega_t$:

$$E[G_t\omega_t \circ k_t] = E[G_t\left(1-\lambda G_t\right)^{-1}\left(\beta_1\iota + \rho\omega_{t-1} + \boldsymbol{x_{t-1}}\boldsymbol{\beta_x} + G_t\boldsymbol{x_{t-1}}\boldsymbol{\beta_{\bar{x}}} + \zeta_t\right) \circ k_t]$$

where $\circ$ is the Hadamard product.[16] Even though capital stock was determined in the previous period, it is still correlated with current productivity spillovers because productivity persists over time, and investment in the previous period was a function of productivity at the time. That is $k_(it) = \kappa(k_{t-1}, i_{t-1}(\omega_{t-1}))$ and therefore, $E[G_t\omega_t \times k_t] \neq 0$. The same argument can be made for labor which is a function of productivity in the same period: $l_{t-1}(\omega_{t-1}) \implies E[G_t\omega_t \circ \ell_{t-1}] \neq 0$.

The direction of bias will depend on the sign and size of λ and the relationship between capital, labor and productivity. For example, if networks generate positive productivity externalities and capital stock is increasing in productivity, then α_k will

[14]Here, I assume that G_t, $\{\boldsymbol{\omega_{jt-1}}\}_{j\in N_{it}}$ and $\{\boldsymbol{x_{jt-1}}\}_{j\in N_{it}}$ are in firm i's information set at the beginning of the period. I discuss this assumption explicitly in the next section.

[15]For example, as De Loecker (2013) notes, including firm's current export status would not be valid because that is dependent on productivity in the same period, but using previous export status would satisfy this condition.

[16]Element-wise multiplication.

be biased upwards. If λ is small enough, then the size of bias will be minimal. TFP values will be underestimated but the direction of bias on λ is unclear.

On their own, correlated effects or network fixed effects do not introduce bias in the estimation of α_k and α_ℓ. Since the common component shocks are idiosyncratic each period, then k_t and ℓ_{t-1}, which were determined in the previous period are independent of c_{ψ_t}. However, to the extent that network components and links do not vary much over time, failing to account for $c_{\psi_t t}$ would bias α_k and α_ℓ estimates.

To illustrate the bias from ignoring endogenous network effects, consider the following process:

$$\omega_t = \rho(I - \lambda G_t)^{-1}\omega_{t-1} + (I - \lambda G_t)^{-1}\zeta_{it} = \rho \sum_{s=0}^{\infty} \lambda^s G_t^s \omega_{t-1} + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_{it} \tag{15}$$

Then the second stage of ACF is equivalent to estimating:[17]

$$\implies y_t = \alpha_\ell \ell_t + \alpha_k k_t + \rho \sum_{s=0}^{\infty} \lambda^s G_t^s \left(y_{t-1} - \alpha_\ell \ell_{t-1} - \alpha_k k_{t-1} - u_{t-1}\right) + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \varepsilon_t$$

Let $\Delta^G x_t = x_t - \rho \sum_{s=0}^{\infty} \lambda^s G_t^s x_{t-1}$, $\Delta_{x_t}^{err} = \rho \sum_{s=1}^{\infty} \lambda^s G_t^s x_{t-1}$ and $\Delta x_t = x_t - \rho x_{t-1} = \Delta^G x_t + \Delta_{x_t}^{err}$. This implies:

$$\Delta^G y_t = \alpha_\ell \Delta^G \ell_t + \alpha_k \Delta^G k_t + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \Delta^G \varepsilon_t \tag{16}$$

This is equivalent to the dynamic panel approach in Blundell and Bond (2000). However, growth in output, labor and capital have been purged of the variation from network effects in the previous period. When we assume no spillovers, we estimate:

$$\Delta y_t = \alpha_\ell \Delta \ell_t + \alpha_k \Delta k_t + u_t \tag{17}$$

Therefore, in the linear AR1 case, ignoring spillovers is equivalent to introducing non-classical measurement error into both output and inputs. Bias from ignoring spillovers can also be characterized as an omitted variables problem. By estimating equation (17), where $u_t = \rho \sum_{s=1}^{\infty} \lambda^s G_t^s \omega_{t-1} + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \varepsilon_t$. That is, the standard ACF procedure succeeds in eliminating the endogeneity problem that arises from input

[17] See section A for derivation.

decisions depending on the firm's own productivity, but is unable to account for the influence of its network's past productivity. In either case, an instrumental variable approach would help to eliminate the problem. The key would be to find variables that are correlated with changes to labor and capital but uncorrelated with output, particularly the input choices and output of other firms.

In the OP/LP case where the labor elasticity is consistently estimated in the first stage, the second stage is equivalent to estimating:

$$\Delta^G \widetilde{y}_t = \alpha_k \Delta^G k_t + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \Delta^G \varepsilon_t \tag{18}$$

$$\tag{19}$$

where $\widetilde{y}_t = y_t - \widehat{\alpha}_\ell \ell_t$. Then by estimating $\Delta\widetilde{y}_t = \alpha_k \Delta k_t + u_t$ under the standard assumption of no-spillovers:

$$plim\ \widehat{\alpha}_k = \frac{cov(\Delta k_t, \Delta\widetilde{y}_t)}{var(\Delta k_t)} \tag{20}$$

$$= \alpha_k \left(1 - \rho \sum_{s=1}^{\infty} \lambda^s \frac{cov(\Delta k_t, G_t^s k_{t-1})}{var(\Delta k_t)}\right) + \rho \sum_{s=1}^{\infty} \lambda^s \frac{cov(\Delta k_t, G_t^s \widetilde{y}_{t-1})}{var(\Delta k_t)} \tag{21}$$

On one hand, α_k is re-scaled by the covariance between the firm's capital growth and its network's previous capital. If this covariance is positive, then it would shrink $\widehat{\alpha}_k$ or even reverse its sign. Higher ρ will increase the attenuation factor, as will λ if it is positive. When λ is negative, it leads to an alternating series that dampens attentuation. The network structure also plays a role: when long chains exist, $G_t^s k_{t-1} > 0$ even for high values of s. By contrast, a network in which firms are paired off, so that the longest chain has length 1. Then $G_t^s k_{t-1} = 0$ for all $s > 1$ and attentuation would be lower under this scenario.

On the other hand, there is another source of bias that depends on the covariance between the firm's capital growth and its network's previous output purged of the variation from labor. When this covariance is positive, $\widehat{\alpha}_k$ overestimates α_k, and the effects of ρ, λ and G_t now work in the opposite direction. Depending on the signs and magnitudes of these covariances, it is possible to obtain estimates of α_k close to the true value if the two opposing effects cancel out.

Even in this simplified setting, the direction and magnitude of bias are not easily

predictable *ex-ante*. This means that one cannot merely apply a bias correction to estimates obtained under standard assumptions. It motivates a modification to the estimation procedure that can flexibly account for a variety of productivity processes and network effects. I propose a modification to the ACF procedure that achieves this without many additional assumptions.

4 Accounting for Spillovers

4.1 Endogenous and Contextual Effects

Assuming network exogeneity and no correlated effects, I write a more general form of the linear-in-means equation (12) above:

$$\omega_t = h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \lambda G_t\omega_t + \zeta_t \tag{22}$$

Note that $h(\cdot)$ is unknown and can be estimated using a polynomial approximation. This allows for flexible interactions between ω_{t-1}, $\boldsymbol{x_{t-1}}$ and $G_t\boldsymbol{x_{t-1}}$. The key requirement is that the endogenous effect enters linearly. This leads to the reduced form:

$$\omega_t = (I - \lambda G_t)^{-1}h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + (I - \lambda G_t)^{-1}\zeta_t \tag{23}$$

$|\lambda| < 1$ implies that we can approximate $(I - \lambda G_t)^{-1}$ by a geometric series.

$$\omega_t = \sum_{s=0}^{\infty} \lambda^s G_t^s h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t \tag{24}$$

This yields a consistent estimate of the conditional expectation of TFP:

$$E[\omega_t|\mathcal{I}_{t-1}] = \sum_{s=0}^{\infty} \lambda^s G_t^s h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) \tag{25}$$

since the resulting error term satisfies the mean independence condition:

$$E\left[(I - \lambda G_t)^{-1}\zeta_t \middle| \mathcal{I}_{t-1}\right] = E\left[\sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t \middle| \mathcal{I}_{t-1}\right] = 0$$

.

Note that equation (24) also indicates how λ can be identified. Given the reduced-

form equation, $G_t\omega_t$ can be written as:

$$G_t\omega_t = G_t h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \sum_{s=1}^{\infty} \lambda^s G_t^{s+1} h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \sum_{s=0}^{\infty} \lambda^s G_t^{s+1}\zeta_t \tag{26}$$

As long as productivity is sufficiently persistent, we can use the current network's past productivity $G_t\omega_{t-1}$ as an instrument for the impact of network's current productivity $G_t\omega_t$. This is because a firm is only affected by its current neighbors' past productivity through the neighbors' current productivity. Therefore, λ is identified from the variation in $G_t\omega_t$.

Equation (26) indicates that there are additional instruments available to identify the endogenous network effect. These are more common in the network effects literature and rely on the existence of intransitive triads in the network (Lee, 2003; Bramoullé et al., 2009). For example $G_t^2\omega_t$ and $G_t^2\boldsymbol{x_{t-1}}$ is one set of possible instruments because G_t^2 captures the neighbors of a firm's neighbors, and these indirect connections affect the firm only through the firm's direct relationships.

Note however, that the relevance of these additional instruments relies on the strength of the endogenous effect. Whereas $G_t\omega_{t-1}$ is a good instrument as long as productivity is persistent, $G_t^2\omega_{t-1}$ requires both persistence and $|\lambda| > 0$ while $G_t^2\boldsymbol{x_{t-1}}$ requires that both endogenous and contextual network effects be nonzero.

Substituting the reduced form equation into the vectorized production function:

$$y_t = \alpha_k k_t + \alpha_\ell \ell_t + (1 - \lambda G_t)^{-1} \left[h(\varphi_{t-1} - \alpha_k k_{t-1} - \alpha_\ell \ell_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \zeta_t\right] + \varepsilon_t \tag{27}$$

which leads to the polynomial expansion:

$$y_t = \alpha_k k_t + \alpha_\ell \ell_t + \sum_{s=0}^{\infty} \lambda^s G_t^s h(\varphi_{t-1} - \alpha_k k_{t-1} - \alpha_\ell \ell_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \varepsilon_t \tag{28}$$

Accounting for network effects in the estimation procedure comes at the cost of additional assumptions. The first, as seen above, is that the endogenous effect enters the productivity process linearly. This would not hold if there were non-monoticities in spillovers. For instance, if firms are likely to free-ride on very productive neigh-

bors and are also negatively affected by very unproductive networks, but are able to learn from moderately productive firms, then the linearity assumption would not hold. However, there is reason to believe that linearity is, at the very least, a good approximation for understanding the network effect and it is a common assumption in the peer effects literature. Furthermore, one need not assume linearity if endogenous spillovers are not contemporaneous. For example, if we assume firms are affected by the past productivity of the previous network ($G_{t-1}\omega_{t-1}$), or the past productivity of their current network ($G_t\omega_{t-1}$), then either of these terms could enter $h(\cdot)$ non-linearly without posing a problem for identification.

Secondly, we need to assume that $\{G_{i,jt}\}_{j\in N_{it}}$ is in the firm's information set $\mathcal{I}_{it-1}$ at the beginning of the period. This is consistent with a network that is fixed over time: $G_t = G \; \forall \; t = 1...T$ or any network formation processes that takes place at the beginning of every period before productivity is realized. For example, in the context of production networks, if all firms choose their suppliers at the beginning of each year, this condition would be met. The key here is the timing: firms make production decisions based on their realized productivities inclusive of spillovers. In addition, $\omega_{jt-1}, \boldsymbol{x_{jt-1}} \in \mathcal{I}_{it-1} \forall j \in N_{it}$. That is, firms can observe the past productivity and decisions of their neighbors. This likely holds true for buyer-supplier relationships in which buyers often do due diligence on future suppliers, and would need to be examined in other contexts such as geographic proximity, family networks, affiliate relationships, interlocking boards, and so on.

Third, I assume that correlations between the TFPs of connected firms are generated by spillovers rather than common shocks. I relax this assumption in the next section.

Finally, this procedure requires that G_t is exogenous, that is, network formation and productivity are not driven by factors that firms observe but we do not. This assumption can also be relaxed but will require the network formation process to be specified. I do so in section 5.

4.2 Correlated Effects

Although network fixed effects alone do not bias the estimates of capital and labor elasticities, if endogenous or contextual spillovers are also present, failing to account for common shocks will lead to the mismeasurement of TFP. Therefore, given a pro-

ductivity process with a component-year-specific fixed effect:

$$\omega_t = h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \lambda G_t\omega_t + c_{\psi_t} + \zeta_t \tag{29}$$

c_{ψ_t} can be eliminated by differencing using a matrix J_t such that $J_t)c_{\psi_t} = 0$. Bramoullé et al. (2009) suggest two ways to define J_t. The first is *within local differencing* by setting $J_t = I - G_t$. This subtracts the mean of a firm's neighbors' variables from the its own. An alternative would be *global differencing*, which subtracts not just the mean of a firm's neighbors, but all the firms in the component. That is, define J_t such that $H_{ij,t} = 1 - \frac{1}{n_{\psi_t}}$ if $i, j \in \psi_t$ and 1 otherwise.

Local differencing would suffice in an undirected network because if two firms are linked, then the link is reported in $G_{ij,t}$ and $G_{ji,t}$. However in directed networks, there may be some firms that are in the same sub-component and are therefore facing component-specific shocks but $\sum_{j \in N_{it}} G_{ij,t} = 0$, because the firm only has connections coming from one direction. For example, in a study of how customers affect the productivity of their suppliers, firm i may be a final goods producer whose productivity generates upstream spillovers but does not supply to any downstream firms. Yet it would be exposed to any shocks that affects the entire supply chain. If edges in G_t are classified as links from suppliers to customers, $G_{ij,t} = 0 \; \forall j$ and $(I - G_t)c_{\psi_t} = c_{\psi_t}$. In this case, local differencing would not eliminate the correlated effect, but global differencing would.

When J_t is chosen appropriately, then transforming equation (29) yields:

$$J_t\omega_t = J_t h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \lambda J_t G_t\omega_t + J_t\zeta_t$$

with the corresponding reduced form:

$$J_t G_t\omega_t = \sum_{s=0}^{\infty} \lambda^s J_t G_t^s h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \sum_{s=0}^{\infty} \lambda^s J_t G_t^s \zeta_t$$

Note that differencing the productivity process will require that the production

function be transformed as well. That is:

$$J_t y_t = \alpha_k J_t k_t + \alpha_\ell J_t \ell_t + \sum_{s=0}^{\infty} \lambda^s J_t G_t^s h(\varphi_{t-1} - \alpha_k k_{t-1} - \alpha_\ell \ell_{t-1}, \boldsymbol{x_{t-1}}, G_t \boldsymbol{x_{t-1}}) + \sum_{s=0}^{\infty} \lambda^s J_t G_t^s \zeta_t + J_t \varepsilon_t \tag{30}$$

4.3 Estimation Procedure

I summarize my benchmark estimation procedure and outline modifications to deal with correlated effects. Estimation is a two-stage process. The first stage is the same as in Ackerberg et al. (2015). Estimate equation (6): $y_t = \alpha_k k_t + \alpha_\ell \ell_t + \mathbb{M}^{-1}(k_t, \ell_t, m_t) + \varepsilon_t$, using a polynomial approximation.[18] This yields estimates $\widehat{\varphi}_t = y_t - \widehat{\varepsilon}_t$.

In the second stage, estimate equation (28) by GMM with $k_t, \ell_t, \widehat{\varphi}_{t-1}, G_t\widehat{\varphi}_{t-1}$ as instruments. Alternatively, to reduce computational complexity, one can concentrate out the parameters in $h(\cdot)$ and proceed as follows. Start with guesses of the production function elasticities: $\alpha_k^*, \alpha_\ell^*$ and compute $\omega_t^* = \widehat{\varphi}_t - \alpha_k^* k_t - \alpha_\ell^* \ell_t$. Estimate the productivity process by 2SLS:

$$\omega_t^* = h(\omega_{t-1}^*, \boldsymbol{x_{t-1}}, G_t \boldsymbol{x_{t-1}}) + \lambda G_t \omega_t^* + u_t \tag{31}$$

with a polynomial approximation of $h(.)$ and $[G_t\omega_{t-1}, G_t^2\omega_{t-1}, G_t^2\boldsymbol{x_{t-1}}]$ as instruments for $G_t\omega_t$. Using predicted values, $E[\omega_t^*|\mathcal{I}_{t-1}]$ from the regression, compute the residual in the productivity process:

$$u_t^* = \omega_t^* - h^*(\omega_{t-1}^*, \boldsymbol{x_{t-1}}, G_t \boldsymbol{x_{t-1}}) - \lambda^* G_t \omega_t^*$$

Then solve for a new set of $(\alpha_k^*, \alpha_\ell^*)$ that satisfy the sample moment conditions:

$$E_{nt}[u_t^* \circ k_t, \ell_{t-1}] = 0 \tag{32}$$

Iterate through all steps of the second stage until the parameters converge to values $[\widehat{\alpha}_1, \widehat{\alpha}_k, \widehat{\alpha}_l]$. The corresponding second stage parameters, $\widehat{\lambda}$ and the parameters in

[18] Like ACF, this estimation procedure can be used with other value-added production function specifications such as the translog.

$\widehat{h}(\cdot)$ are consistent estimates of network effects. Standard errors can be obtained by residual-based or vertex bootstrapping. See section **??** in the appendix for details on bootstrapping network data.

To account for correlated effects, estimate the first stage as in the benchmark procedure, and apply the J_t transformation to all variables in the second stage.

5 Network Endogeneity

So far, I have assumed that the network is exogenous, but it is also possible that a firm's productivity may be correlated with how it forms relationships. This issue is reminiscent of the selection problem in Olley and Pakes (1996) – firms are only observed if their productivity is above some threshold. In this case, observed interfirm relationships may depend on TFP. To address this issue, I incorporate the network selection model in Arduini et al. (2015) and Qu et al. (2017) into the benchmark estimation procedure above.

5.1 Network Selection Model

Endogenous network formation as modeled by Qu et al. (2017) and Arduini et al. (2015) highlights a possible link between a firm's TFP and the nature of its network. Shocks to productivity are correlated with the chances of meeting potential partners. For example, firms that are better able to search for buyers or suppliers may also be more productive. In this case, a positive relationship between a firm's TFP and its networks' TFP or choices would be a result of the improved search rather than any spillovers.[19]

At the beginning of each period, firms i and j consider the surplus of a link

[19]Other studies such as Goldsmith-Pinkham and Imbens (2013) and Hsieh and Lee (2016) model network endogeneity as a correlation between unobserved variables in the network selection model and the error term of the outcome equation. The interpretation differs; in this setting selection would be driven by unobserved synergies such as common business philosophies. If these factors are also correlated with productivity, then estimated spillovers would capture the effect of assortativity in these unobserved characteristics (see Serpa and Krishnan (2018)) for an application to productivity spillovers. I choose the Arduini et al. (2015) model for two reasons. First, it allows me to explicitly highlight the dual role that productivity may play in search and interfirm spillovers. Secondly, the reduction of the problem to a selection correction term preserves the usual structure of the estimator, while the Goldsmith-Pinkham and Imbens (2013) and Hsieh and Lee (2016) relies on Bayesian estimation of a full likelihood model.

$V_i(A_{ij,t})$. Both firms want to form a link if $V_i(A_{ij,t} = 1) - V_i(A_{ij,t} = 0) > 0$.[20] I parametrize this difference in surplus as:

$$V_i(A_{ij,t} = 1) - V_i(A_{ij,t} = 0) = U_{ijt}(\gamma) + \xi_{ijt}$$

where ξ_{ijt} is i.i.d and follows a logistic distribution.

$$U_{ijt}(\gamma) = \gamma_1 + \boldsymbol{z_{it}\gamma_i} + \boldsymbol{z_{jt}\gamma_j} + \boldsymbol{z_{ijt}\gamma_{ij}} + \gamma_h H_{ijt} \tag{33}$$

Note that despite the slight abuse of notation, $\boldsymbol{\gamma_i, \gamma_j, \gamma_{ij}}$ are not random coefficients. They are parameters whose subscripts denote that they correspond to i, j or the dyad's characteristics.

$\boldsymbol{z_{it}}$ may include ω_{it-1}, x_{it-1} and other variables such as industry that influence a firm's relationship decision but may have no direct impact on productivity. $\boldsymbol{z_{ijt}}$ usually includes the distance between i and j's characteristics: $|\boldsymbol{z_{it}} - \boldsymbol{z_{jt}}|$ or some other dyad-specific measures, such as the physical distance between the firms, industry input-output shares, etc. A negative coefficient on $|\boldsymbol{z_{it}} - \boldsymbol{z_{jt}}|$ indicates that firm i wants to match with firms that are similar. H_{ijt} measures past linkages; a large and positive γ_h indicates that firm i prefers to stick with its previous partners. Past linkages can be specified broadly; for instance, $H_{ijt} = A_{ij,t-1}$ would mean that firm i only considers linkages from the previous period, while $H_{ijt} = \mathbb{1}\left(\sum_{s=1}^{m} A_{ij,t-s} > 0,\ m \leq t\right)$ measures whether i and j were connected in any of the last m periods.[21]

The probability that a link $A_{ij,t}$ forms is given by:

$$P(A_{ij,t} = 1 \mid \mathcal{I}_{t-1}) = P(U_{ijt}(\gamma) + \xi_{ijt} > 0) = \frac{e^{U_{ijt}(\gamma)}}{1 + e^{U_{ijt}(\gamma)}}$$

The specified model, coupled with a logistic distribution implies that, conditional on firm and dyad characteristics, historical connectivity, and the unobserved ξ_t, the probability that i wants to form a link with j is independent of its decision to connect with some other firm k. While this may be restrictive, it is analytically and computationally tractable, and still manages to capture important features of real-world

[20] This model can apply to both directed and undirected networks. For example, in a buyer-supply network, the the surplus from i supplying j would be considered differently from the reverse direction.

[21] There are alternative models such as Graham (2017) that include firm-year fixed effects in the dyadic regression model. Estimation of such models will depend on the sparsity of the network.

networks.

For example, this model allows for the possibility that a firm can choose multiple partners; i need not prefer j to all other firms, it just needs to prefer matching with j to not matching. This is useful for characterizing production networks, in which a non-negligible number of firms trade with more than one partner. As in Goldsmith-Pinkham and Imbens (2013), this model can also accommodate some interdependence in the linking decision through the choice of variables such as the number of links in the previous period, whether the firms had neighbors in common etc.

Network endogeneity arises from the relationship between ξ_{ijt} and the error term in the productivity process, ζ_{it}. Let $\xi_{it}^{'} = \{\xi_{ijt}\}_{j\neq i}^{n_t}$ be a row vector of the error terms from all the dyadic regressions with links originating from i. $(\zeta_{it}, \xi_{it}^{'}) \sim i.i.d.(0, \Sigma_{\zeta\xi})$ where $\Sigma_{\zeta\xi} = \begin{pmatrix} \sigma_\zeta^2 & \sigma_{\zeta\xi}^{'} \\ \sigma_{\zeta\xi} & \Sigma_\xi \end{pmatrix}$ is positive definite, σ_ζ^2 is a scalar, $\sigma_{\zeta\xi}$ is an $n_t - 1$ column vector of covariances, and $\Sigma_\xi = \sigma_\xi^2 I_{n_t-1}$. Stacking all the ξ_{it}'s in a matrix:

$$\Xi_t = \begin{bmatrix} \xi_{1t}^{'} \\ \vdots \\ \xi_{n_t t}^{'} \end{bmatrix}$$

then the error term in the productivity process can be written as:

$$\zeta_{it} = \Xi_t \boldsymbol{\delta} + \nu_t$$

where $\boldsymbol{\delta} = \Sigma_\xi^{-1}\sigma_{\zeta\xi}$, ν_t is independent of ξ_{it} and $\sigma_\nu^2 = \sigma_\zeta^2 - \sigma_{\zeta\xi}^{'}\Sigma_\xi^{-1}\sigma_{\zeta\xi}$. Therefore, the productivity process becomes:

$$\omega_t = h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \lambda G_t\omega_t + \Xi_t\boldsymbol{\delta} + \nu_t \tag{34}$$

G_t is endogenous when $\sigma_{\zeta\xi} \neq 0$ and the selectivity bias is equal to $\Xi_t\boldsymbol{\delta}$.

5.2 Accounting for Selection

To the estimate model, assume ζ_{it} is normally distributed. Then Arduini et al. (2015) shows that the selectivity bias can be controlled for using a Heckman-type mills ratio:

$$\begin{aligned}\mu_{it} &= \sum_{j\neq i}^{N_t} g_{ij,t}\frac{\phi(\Phi^{-1}(p))}{\Phi(\Phi^{-1}(p))} + (1-g_{ij,t})\frac{\phi(\Phi^{-1}(p))}{1-\Phi(\Phi^{-1}(p))} \\ &= \sum_{j\neq i}^{N_t} g_{ij,t}\frac{\phi(\Phi^{-1}(p))}{p} + (1-g_{ij,t})\frac{\phi(\Phi^{-1}(p))}{1-p}\end{aligned} \tag{35}$$

where $p = P(A_{ij,t} = 1 \,|\, \mathcal{I}_{t-1})$, and ϕ and Φ are the probability and cumulative density functions for a standard normal variable. The i.i.d assumption on ξ_{ijt}'s dispenses with the need to estimate all $N_t - 1$ parameters in $\boldsymbol{\delta}$. Instead, due to the summation above, one only has to estimate a single parameter $\delta = \frac{\sigma_{\zeta\xi}}{\sigma_\xi^2}$.

5.3 Estimation Procedure

Incorporating the selection model is similar to the Olley and Pakes (1996) correction for attrition. The first stage of my benchmark procedure is unchanged with the estimation of $\widehat{\varphi}_{it}$ and $\widehat{\varepsilon}_{it}$ using the proxy variable. In the second stage, starting with the initial guesses of the labor and capital coefficients $(\alpha_k^*, \alpha_\ell^*)$, compute $\omega_{it-1}^* = \widehat{\varphi}_{it-1} - \alpha_k^* k_{it-1} - \alpha_\ell^* \ell_{it-1}$.

Using ω_{it-1}^* and other variables that could determine the observed links between firms, estimate the selection model in equation (33) to obtain γ^*. Next, compute the predicted probabilities $p^* = \frac{e^{U_{ijt}(\gamma^*)}}{1+e^{U_{ijt}(\gamma^*)}}$ and the selection correction term $\mu_{it}^* = \sum_{j\neq i}^{N_t} g_{ij,t}\frac{\phi(\Phi^{-1}(p^*))}{p^*} + (1-g_{ij,t})\frac{\phi(\Phi^{-1}(p^*))}{1-p^*}$. Include this correction term as one of the explanatory variables in the productivity process equation:

$$\omega_t^* = \sum_{s=0}^{\infty} \lambda^s G_t^s h(\omega_{t-1}^*, \boldsymbol{x_{t-1}}, G_t \boldsymbol{x_{t-1}}) + \delta \sum_{s=0}^{\infty} \lambda^s G_t^s \mu_{it}^* + u_t \tag{36}$$

The resulting residuals are now purged of the omitted variable bias arising from network selection and can be used to construct the sample moments in (31) for identification of the elasticities.[22]

[22] In principle, the selection model would be re-estimated for each value of ω_{it-1}^* as the values $(\alpha_k^*, \alpha_\ell^*)$ are updated in each iteration. However, this significantly increases the computational cost

6 Monte Carlo Experiments

I conduct three sets of experiments to assess the performance of the standard ACF estimator and my modified procedure when various types of network effects are present. In the first set of experiments, I examine how each type of network effect individually affects the bias and efficiency of capital and labor elasticity estimates obtained using the ACF procedure. Next, I demonstrate how my modified procedure performs when endogenous, contextual and correlated effects are cumulatively present and consider the sensitivity of the estimates from my benchmark procedure to misspecification. Finally, I compare the performance of ACF and my benchmark procedure as the size of the endogenous effect, the persistence of productivity and the density of the network vary.

For all three experiments, I draw a balanced sample of 1000 firms over 10 years. I use a Cobb-Douglas production function in logs:

$$y_{it} = \alpha_\ell \ell_{it} + \alpha_k k_{it} + \omega_{it} + \varepsilon_{it}$$

where $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. I set $\alpha_\ell = 0.6, \alpha_k = 0.4$ and $\sigma_\varepsilon^2 = 1$.[23] The productivity process varies depending on the experiment. To avoid the impact of arbitrary initial values, I simulate 20 periods and discard the first 10.

To induce variation in cluster (component) size and the length of supply chains, I split the firms into four industries with 400, 300, 200, and 100 firms in the first, second, third and fourth industries respectively and construct an inter-industry trade structure as follows: Industry 1 sells 17, 33 and 44 percent of its output to industries 2, 3 and 4 respectively. 2 sells to 50 percent each to 3 and 4, while industry 3 sells all its output to 4. The fourth industry sells nothing to other firms. This structure is fixed over time, and does not represent the actual network but is a measure of industry compatibility that I use to generate both exogenous and endogenous networks as described below.[24]

of the procedure. As long as the initial guesses of the elasticities, such as those obtained from an OLS regression, are reasonably close to their true values measurement error in the lagged TFP variable should not have an outsized effect on the estimates of the selection correction term. In my Monte Carlo simulations, results were quite similar when selection was estimated only once and when it was re-estimated in each iteration.

[23] See section C for further details on the Monte Carlo setup.

7 Extensions

7.1 Gross Production Functions

So far, I have only considered a structural value-added production function, which often requires the assumption that the production function is Leontief with respect to intermediate inputs. In this section I consider a framework exploiting first order conditions on intermediate input choices as in Gandhi, Navarro, and Rivers (2020, GNR hereafter). Under similar assumptions as in the proxy variable approach above, the standard GNR procedure can be modified to jointly estimate network effects and productivity.

Like ACF, the GNR methodology assumes that TFP enters the production function in a Hicks-neutral fashion. However, intermediate inputs now enter directly into the production function:

$$\begin{aligned} Y_t &= F(L_t, K_t, M_t)e^{\omega_t+\varepsilon_t} \\ \iff y_t &= f(\ell_t, k_t, m_t) + \omega_t + \varepsilon_t \end{aligned} \tag{37}$$

For simplicity, assume that materials are flexible while both labor and capital have dynamic implications.

The procedure consists of two stages. The first stage exploits first order conditions from profit maximization to estimate the elasticity of intermediate inputs with respect to output. Given the production technology above, the firm chooses materials to maximize profits:

$$\max_{M_t} P_t E[F(L_t, K_t, M_t)e^{\omega_t+\varepsilon_t}] - P_t^M M_t \tag{38}$$

where P_t and P_t^M are the prices of output and materials respectively. The static first order condition with respect to materials is:

$$P_t \frac{\partial}{\partial M_t} F(L_t, K_t, M_t)e^{\omega_t}\mathcal{E} = P_t^M \tag{39}$$

where $\mathcal{E} \equiv E[e^{\varepsilon_t} \,|\, \mathcal{I}_t] = E[e^{\varepsilon_t}]$ which relies on the assumption that the error terms are unconditionally independent.[25]

[25] See Gandhi et al. (2020) for details on estimation under a relaxed conditional independence

It is also pertinent to note that this first order condition makes an implicit assumption about market structure: that the firm is a price-taker in both input and output markets. Therefore, this framework cannot directly examine impacts of or effects on market power. I retain this assumption in my modified procedure.

$$\begin{aligned}
&\frac{\partial}{\partial M_t} F(L_t, K_t, M_t) e^{\omega_t} \mathcal{E} = \frac{P_t^M}{P_t} \\
&\frac{M_t}{Y_t} \frac{\partial}{\partial M_t} F(L_t, K_t, M_t) e^{\omega_t} \mathcal{E} = \frac{P_t^M M_t}{P_t Y_t} \\
&\ln\left(\frac{\partial}{\partial m_t} f(\ell_t, k_t, m_t)\right) - \varepsilon_t + \ln(\mathcal{E}) = s_t
\end{aligned} \tag{40}$$

where $s_t \equiv \ln(\frac{P_t^M M_t}{P_t Y_t})$ is the log of the intermediate input expenditure share of revenue.

$$E[\varepsilon_t \mid \mathcal{I}_t] = 0 \implies E[s_t \mid \mathcal{I}_t] = \ln\left(\frac{\partial}{\partial m_t} f(\ell_t, k_t, m_t)\right) + \ln(\mathcal{E}) \tag{41}$$

Let $D^{\mathcal{E}}(\ell_t, k_t, m_t) \equiv \dfrac{\partial}{\partial m_t} f(\ell_t, k_t, m_t) \times \mathcal{E}$. Then given the moment of ε_t in (41) above, $\ln D^{\mathcal{E}}(\ell_t, k_t, m_t)$ can be estimated by non-linear least squares regression of the materials expenditure share on the log of a polynomial in labor, capital and materials. Furthermore:

$$\begin{aligned}
\varepsilon_t = \ln D^{\mathcal{E}}(\ell_t, k_t, m_t) - s_t \implies e^{\varepsilon_t} &= D^{\mathcal{E}}(\ell_t, k_t, m_t) e^{-s_t} \\
\mathcal{E} = E[e^{\varepsilon_t}] &= E[D^{\mathcal{E}}(\ell_t, k_t, m_t) e^{-s_t}]
\end{aligned} \tag{42}$$

Using the estimates of $D^{\mathcal{E}}$ from the share regression, we can replace the moment in (42) with its empirical equivalent and compute the constant $\mathcal{E}$. This enables us obtain an estimate of the materials elasticity:

$$D(\ell_t, k_t, m_t) = \frac{\partial}{\partial m_t} f(\ell_t, k_t, m_t) = \frac{D^{\mathcal{E}}(\ell_t, k_t, m_t)}{\mathcal{E}} \tag{43}$$

The second stage of GNR relies further assumptions on the productivity process to

assumption.

estimate the rest of the production function. By the fundamental theorem of calculus:

$$\int \frac{\partial}{\partial m_t} f(\ell_t, k_t, m_t) dm_t = f(\ell_t, k_t, m_t) + \mathcal{C}(\ell_t, k_t) \tag{44}$$

The goal is to estimate $\mathcal{C}(\cdot)$ since we can compute $\int \frac{\partial}{\partial m_t} f(\ell_t, k_t, m_t) dm_t$ using $D(\ell_t, k_t, m_t)$ from the first stage. By substituting for $f(\ell_t, k_t, m_t)$ using equation (37):

$$\begin{aligned} \int \frac{\partial}{\partial m_t} f(\ell_t, k_t, m_t) dm_t &= y_t - \omega_t - \varepsilon_t + \mathcal{C}(\ell_t, k_t) \\ \mathcal{Y}_t \equiv y_t - \int \frac{\partial}{\partial m_t} f(\ell_t, k_t, m_t) dm_t - \varepsilon_t &= -\mathcal{C}(\ell_t, k_t) + \omega_t \end{aligned} \tag{45}$$

It is at this point that the assumption on the productivity evolution process comes into play. GNR maintains the same first-order Markov assumption as ACF:

$$\omega_t = h(\omega_{t-1}) + \eta_t, \qquad \text{where } E[\eta_t \mid \mathcal{I}_{t-1}] = 0 \tag{46}$$

$$\begin{aligned} \omega_{t-1} &= \mathcal{Y}_{t-1} + \mathcal{C}(\ell_{t-1}, k_{t-1}) \\ \implies \mathcal{Y}_t &= -\mathcal{C}(\ell_t, k_t) + h(\mathcal{Y}_{t-1} + \mathcal{C}(\ell_{t-1}, k_{t-1})) + \eta_t \end{aligned} \tag{47}$$

We can estimate $\mathcal{C}(\cdot)$ and $h(\cdot)$, normalizing the former to contain no constant, based on unconditional moments derived from $E[\eta_t \mid \mathcal{I}_t$:

$$\begin{aligned} E[\eta_t \ell_t^{\tau_\ell} k_t^{\tau_k}] &= 0 \\ E[\eta_t \mathcal{Y}_{t-1}^{\tau_\mathcal{Y}}] &= 0 \end{aligned} \tag{48}$$

where τ_ℓ, τ_k and $\tau_\mathcal{Y}$ are determined by the degrees of the polynomial approximations for $\mathcal{C}(\cdot)$ and $h(\cdot)$ respectively.

7.1.1 Accounting for Network Effects

As with the modified ACF approach, I maintain the same assumptions and procedure in the first stage of GNR. Network effects come into play at the second stage when the law of motion on productivity is required for identification.

Note however, that by maintaining the same assumptions in the first stage, I do not account for ways in which the firm's network could potentially influence its intermediate input choices. For now, I focus specifically on network effects that operate

through productivity spillovers and leave the implications for materials demand for future work.

I replace the productivity evolution process in (46) with one that allows for a linearly additive endogenous network effect:[26]

$$\begin{aligned} & \omega_t = h(\omega_{t-1}) + \lambda G_t \omega_t + \zeta_t \qquad \text{where } E[\zeta_t \mid \mathcal{I}_{t-1}] = 0 \\ \implies & \omega_t = \sum_{s=0}^{\infty} \lambda^s G_t^s h(\omega_{t-1}) + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t \end{aligned}$$

The equation (47) becomes:

$$\mathcal{Y}_t = -\mathcal{C}(\ell_t, k_t) + \sum_{s=0}^{\infty} \lambda^s G_t^s h(\mathcal{Y}_{t-1} + \mathcal{C}(\ell_{t-1}, k_{t-1})) + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t \tag{49}$$

This yields an additional set of moments from which the endogenous effect λ can be identified:

$$E[\zeta_t G_t^s \mathcal{Y}_{t-1}^{\tau_{\mathcal{Y}}}] = 0 \qquad \text{where } s \geq 1 \tag{50}$$

7.2 Alternative Network Effect Specifications

The modified ACF procedure introduced in section 4 can accommodate specifications of the productivity process that account for other ways in which spillovers may occur. In this section, I consider some of these specifications, and how they affect the estimator and what additional assumptions are needed, if any.

7.2.1 Local-Aggregate Endogenous Effect

The linear-in-means equation considered so far is also known as the local-average model because it assumes that the average productivity and characteristics of a firm's neighbors is the key source of spillovers. Another model is the local-aggregate model as in Liu and Lee (2010) and Liu et al. (2014), that considers the sum rather than the average. That is:

$$\omega_t = h(\omega_{t-1}, \boldsymbol{x_{t-1}}, A_t \boldsymbol{x_{t-1}}) + \lambda A_t \omega_t + \zeta_t \tag{51}$$

[26] For clarity of exposition, I leave out contextual and correlated effects, but they can be included in much the same way as with ACF.

where A_t is the adjacency matrix. This model has different implications from the local-average model. There are also hybrid models that include local-average contextual effects and local-aggregate endogenous effects:

$$\omega_t = h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \lambda A_t\omega_t + \zeta_t \tag{52}$$

or both local-average and local-aggregate endogenous effects:

$$\omega_t = h(\omega_{t-1}, \boldsymbol{x_{t-1}}, G_t\boldsymbol{x_{t-1}}) + \lambda_A A_t\omega_t + \lambda_G G_t\omega_t + \zeta_t \tag{53}$$

See Liu and Lee (2010) and Liu et al. (2014) for further discussion of the conditions under which these network effects are identified. In general as long as the matrix inversion conditions to obtain a reduced form and the information set conditions hold, my benchmark procedure only needs to be modified by changing the network matrix where necessary.

7.2.2 Heterogeneous Network Effects

So far, my model of network effects has assumed homogeneous spillovers. However, the model can account for a finite set of heterogeneous network effects. If I partition the network into a finite set of B groups such as buyers and suppliers, industries, or based on firm size, then I can estimate:

$$\omega_t = h(\omega_{t-1}, \boldsymbol{x_{t-1}}, \{G_{b,t}\boldsymbol{x_{t-1}}\}_{b=1}^{B}) + \sum_{b=1}^{B} \lambda_b G_{b,t}\omega_t + \zeta_t \tag{54}$$

Note that $\boldsymbol{x_{t-1}}, \{G_{b,t}\boldsymbol{x_{t-1}}\}_{b=1}^{B}\omega_t = \lambda G_t\omega_t$ where λ is a weighted average of the heterogeneous effects. Therefore, my benchmark procedure can still be used to consistently estimate TFP without any modification. Afterwards, the heterogeneous network effect parameters can be obtained using the specification above. Dieye and Fortin (2017) and Patacchini et al. (2017) discuss the identification conditions and estimation procedures for this model in greater detail.

8 Results

In this section, I use my empirical framework to examine the magnitude of endogenous productivity spillovers through vertical relationships in the US production network. I explore how these spillovers vary over time, industry and firm size and document substantial heterogeneity in the sources and recipients of network effects.

I estimate a gross production function with a linear intermediate input share equation and a second-degree polynomial in capital and labor in the second stage.[27] I also estimate a value-added Cobb Douglas production function with materials as the proxy variable and a second-degree polynomial in the first stage. In both specifications, I assume a linear productivity process that includes an endogenous network effect and recover both production function elasticities and productivity spillovers from my modified approach. Because spillovers imply that TFP is jointly determined for linked firms across industries, the production function cannot be estimated separately for each industry. Therefore, I control for industry and year fixed effects in the productivity equation. In addition, due to the observed variation in the network structure over time, I estimate both specifications separately for each decade in the sample.

I compare my estimates with results from standard GNR and ACF approaches with industry and year fixed effects in the productivity equation for comparability. Because standard approaches do not yield estimates of productivity spillovers, I use TFP estimates from these procedures in a second stage. To obtain network effect coefficients, I apply the generalized 2SLS (G2SLS) approach in Lee (2003) and Bramoullé et al. (2009). In the first step, I estimate λ^* by 2SLS using $[G_t\omega_{t-1}, G_t^2\omega_{t-1}]$ as instruments for $G_t\omega_t$. I compute $E^*[G_t\omega_t|\mathcal{I}_{t-1}]$ using the reduced form equation in (13). This is the feasible estimate of the best instrumental variable (IV) for $G_t\omega_t$. Then I estimate 2SLS again, this time with $E^*[G_t\omega_t|\mathcal{I}_{t-1}]$ instrumenting for $G_t\omega_t$. To eliminate component-year fixed effects, I apply global differencing described in section 4.2 to both standard and modified procedures.

As discussed in section 2, the buyer-supplier network is only partially observed because firms only need to report their major customers; only about 18% of links fall below the 10% sales threshold. To address this, I rely on information about link intensity: I weight each relationship by the value traded between the two firms in that year. This mitigates some of the bias from missing links, because links that fall below the 10% threshold are would have weights close to zero. There is also the added advantage of allowing more important trading partners to have a larger impact on a firm's productivity.[28]

[27] This specification implies a translog production function.

[28] As a robustness check, I estimate all specifications with an unweighted network in section F.2 of the appendix. The results are similar in magnitude, indicating that major trading partners are the more salient sources of spillovers.

8.1 Production Function Elasticities

Tables 3 and 4 report the estimated elasticities of output with respect to inputs from gross output and value-added production functions, respectively. GNR/ACF refer to the standard procedures, GNR-N/ACF-N denote my modified approach that accounts for endogenous productivity spillovers, and GNR-ND/ACF-ND indicate specifications that account for both endogenous network effects and component-year fixed effects. Because I assume that the network does not affect intermediate input demand in the gross output specification, the elasticity of output with respect to materials does not vary across specifications.

Estimated capital and labor elasticities are also quite similar with and without accounting for network effects. The relative importance of each input varies over time; in the gross output specification, the elasticity of output with respect to labor falls from about 0.49 between 1977-1986 by 0.26 in the 2007-2016 period. By contrast, results from in the value-added specification move in the opposite direction, with labor elasticity rising from about 0.62 to 0.68 over the same time horizon.

Table 3: Gross Output Production Function Elasticities

Period	Estimator	Capital	Labor	Materials
1977-1986	GNR	0.280	0.489	0.341
	GNR-N	0.271	0.497	0.341
	GNR-ND	0.279	0.493	0.341
1987-1996	GNR	0.267	0.382	0.489
	GNR-N	0.266	0.383	0.490
	GNR-ND	0.271	0.379	0.490
1997-2006	GNR	0.159	0.278	0.529
	GNR-N	0.173	0.269	0.529
	GNR-ND	0.170	0.273	0.529
2007-2016	GNR	0.183	0.263	0.508
	GNR-N	0.194	0.257	0.508
	GNR-ND	0.191	0.262	0.508
All	GNR	0.210	0.321	0.500
	GNR-N	0.219	0.318	0.500
	GNR-ND	0.221	0.317	0.500

This table reports the average input elasticities from a gross output production function estimated on US firms in Compustat. Estimators are based on Gandhi et al. (2020): GNR denotes the standard procedure with a linear first stage, a second-degree polynomial in the second stage, and a linear productivity process. GNR-N and GNR-ND are modifications to accommodate network effects and network differencing respectively. All specifications include industry and year fixed effects in the productivity process.

Table 4: Value-Added Production Function Elasticities

Period	Estimator	Capital	Labor
1977-1986	ACF	0.395	0.632
	ACF-N	0.398	0.629
	ACF-ND	0.405	0.623
1987-1996	ACF	0.441	0.606
	ACF-N	0.437	0.614
	ACF-ND	0.429	0.626
1997-2006	ACF	0.362	0.670
	ACF-N	0.361	0.672
	ACF-ND	0.347	0.685
2007-2016	ACF	0.327	0.670
	ACF-N	0.326	0.671
	ACF-ND	0.316	0.686
All	ACF	0.384	0.644
	ACF-N	0.383	0.646
	ACF-ND	0.375	0.657

This table reports input elasticities of a Cobb-Douglas value-added production function (in logs) estimated on US firms in Compustat. Estimators are based on Ackerberg et al. (2015): ACF denotes the standard procedure with a second-degree polynomial in the first stage and a linear productivity process. ACF-N and ACF-ND are modifications to accommodate network effects and network differencing respectively. All specifications include industry and year fixed effects in the productivity process.

8.2 Endogenous Productivity Spillovers

I now turn to estimates of productivity spillovers. First, I define the network as undirected: a firm j belongs in firm i's neighborhood if it either buys from or sells to the firm. I weight each relationship by the value traded between the two firms in that year.[29] Table 5 and figure 8 show the endogenous network effects from gross output TFP.

Based on TFP measures from the standard GNR approach, the results suggest that a firm's productivity rises by 0.084 percent in the short run when its average buyer or seller gets 10 percent more productive. Accounting for endogenous productivity spillovers in TFP estimation raises the point estimate 0.09 percent, but differencing out common shocks to productivity lowers the estimate to 0.076 percent. The persistence of TFP over time also implies a substantial long-run effect of having more trading partners in one period. An estimated coefficient of 0.9 on ln TFP_{t-1}, means that transacting with a 10 percent more efficient firm in a single period results in a long-run efficiency gain of 0.76 percent. In the value-added specification, the impact of correlated effects is striking; the estimated short-run impact of a 10 percent rise in the average neighbor's TFP goes from 0.07 percent with standard ACF to 0.01 when I account for both endogenous and correlated effects in the production function estimation.

Across all specifications, estimates from the standard approach and my modified procedure are often statistically indistinguishable. This is consistent with the discussion in section A and results from the Monte Carlo experiment in table **??**: standard approaches yielded estimates of productivity spillovers that were closest to the true effect when the network density was between 0.1 and 0.3 percent. As shown in figure 4, the density of the observed network in my sample ranges from 0.12 to 0.28 percent and lies within the region with the least bias in estimated spillovers.

It is worth noting, however, that because these spillovers have a cumulative impact over time and space, even small differences in these point estimates could have result in substantially different implications. To illustrate this, I simulate a growth path for the average firm in 1978 under the assumption that it is connected to the median firm in the same year. For simplicity, I assume there are no shocks to productivity and the average firm is also its partner's only connection, and this relationship remains

[29] As a robustness check, I estimate all specifications with an unweighted network. The results are in section F.2 of the appendix and are qualitatively similar.

the same for all periods. Then I compute:

$$\widetilde{\omega}_t = \left(I - \widehat{\lambda}G\right)^{-1}\left(\widehat{\beta}_1 + \widehat{\rho}\widetilde{\omega}_{t-1}\right), \qquad G = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{55}$$

where $t = \{1979 \ldots 2016\}$, $\widehat{\beta}_1 = 0.203$, and $\widehat{\rho} = 0.9$. $\widehat{\lambda}$ takes on three possible values: the point estimates from GNR, GNR-N and GNR-ND. I difference the cumulative TFP growth from what it would be in a no-spillover scenario in which $\widehat{\lambda} = 0$. Figure 12 shows that an endogenous effect of 0.0076, as obtained from GNR-ND, implies that the average firm would grow an additional 16.3 percent due to spillovers by 2016. Standard GNR would overstate the cumulative impact of spillovers, implying 18 percent additional growth, while accounting for network effects without eliminating common productivity shocks would suggest a cumulative spillover effect of 19.6 percent.

To understand the economic importance of these spillovers in the cross-section, I compute a back-of-the-envelope estimate of the impact of the most connected firm in each year on aggregate TFP through spillovers. Let j denote the most central firm in year t. I sum up j's contribution to the network average for each of its connections, weighting by firm i's nominal revenues in that period, and multiply that by the spillover estimate. That is:

$$\text{Contribution}_{jt} = \widehat{\lambda}\sum_{it} weight_{it} G_{ijt} \tag{56}$$

whew $weight_{it} = \frac{Revenue_{it}}{Avg\ Revenue_t}$. Figure 14 shows that for $\widehat{\lambda} = 0.002$, a 10 percent increase in the TFP of the most central firm would correspond with a 0.25 to 1.9 percent rise in aggregrate TFP through spillovers alone.

8.2.1 Relationship Direction and Dynamics

Next, I examine how spillovers depend on the nature of the relationship between firms. Figure 10 and table 7 show that productive suppliers have almost 5 times the impact on their customers as productive buyers have on their suppliers: a 10 percent more productive supplier raises efficiency by 0.095 percent while customers raise productivity by 0.02 percent. As depicted in figure 13, having a more productive supplier implies that the average firm would grow an additional 19.3 percent over the

sample period due to spillovers, as compared to 4.3 percent from a more productive customer.[30]

To investigate how much this is driven by maintaining old relationships as opposed to forming new ones, I decompose the interaction matrix into buyers/sellers that the firm traded with in both the current and the previous year ($G_t \cap G_{t-1}$), and new links ($G_t \setminus G_{t-1}$). The results in table 9 suggest that both old and new suppliers are important sources of spillovers, while new customers are the primary sources of buyer-to-seller spillovers.

Estimates from value-added specifications do not show a significant difference between spillovers from buyers to sellers or vice versa. In figure 11 and table 8, a 10 percent more efficient supplier is associated with a 0.016 percent rise in productivity while the effect of buyers is 0.011 percent. These effects are statistically indistinguishable from each other. For the rest of this discussion, I focus on estimates from gross output production functions, but additional results from value-added specifications are in section F.1 in the appendix.

8.2.2 Heterogeneity by Sector

In this section, I investigate how the spillovers transmit within and across sectors. I estimate a gross output production function with endogenous and correlated effects, allowing spillovers to vary by the sector of the firm and its trading partners. I classify sectors based on the share of observed links that are from sellers to buyers, or from buyers to sellers respectively. If 50 percent or more of links between sector u and sector v are from suppliers in u to customers in v, then the spillovers from u to v are classified as downstream, while the impact of sector v on firms in u is considered an upstream spillover. Figures 15 and 16 depict estimates of downstream and upstream spillovers, respectively that are significant at the 5 percent level. Table 10 reports the full set of estimates.

These results highlight the important role of information technology (IT), retailers and services in productivity growth within the US. As shown by figure 15, the substantial downstream spillovers occur within the electronics manufacturing sector. Furthermore, the synergies between electronics manufacturing and the finance, insurance and real estate sector is primarily driven by technology patent holders (SIC

[30] Larger confidence intervals in the 1977-1986 period are likely due to a combination of fewer firms in the sample, a denser network, and fewer reported links.

6794 and NAICS 533110) such as InterDigital Inc. which provides mobile technology research services to mobile phone manufacturers such as Apple. Manufacturers also tend to amplify the impact of productivity growth in other sectors because they enjoy efficiency boosts from both directions: electronics manufacturers from mainly customers and manufacturers of non-durables from their suppliers (see figure 16). Retailers are an import source of upstream spillovers, while transport and warehousing firms generate downstream spillovers. None of these sectors receives efficiency boosts from other sectors, and in fact, experience negative network effects. Negative endogenous effects suggests free-riding or, given that I use deflated sales as a measure of output, downward pressure on prices by more profitable firms.

8.2.3 The Role of Firm Size

Finally, I consider the role of firm size in the transmission of efficiency gains through the production network. I classify firms as large if they have 500 or more employees, the definition used by the US BEA. The results are reported in table 11 and figure 17 highlights estimates that are significant at the 5% level. Large productive suppliers are an important source of productivity gains for both large and small firms, with small customers benefiting nearly twice as much as large buyers. Small efficient suppliers also have a substantial impact on their large customers. The effect of customer efficiency on their suppliers is driven solely by large firms.

Given that the average firm in my sample is larger than the average firm in the US, at least 60 percent or more the sample can be classified as large based on this definition. In table 12, I check how sensitive these results are to different classifications of firm size. I consider three alternative definitions based on the number of employees: greater than or equal to 1000, 5000 or an industry-year specific median. The results are similar across definitions except that, as excepted, the impact of large firms diminishes while that of small firms rises.

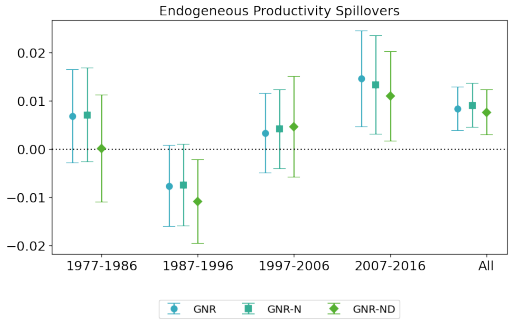


Figure 8: Spillover Estimates (Gross-Output)
This figure shows point estimates and 95% confidence intervals of endogenous productivity spillovers from a gross output production function. See table 5 for standard errors.

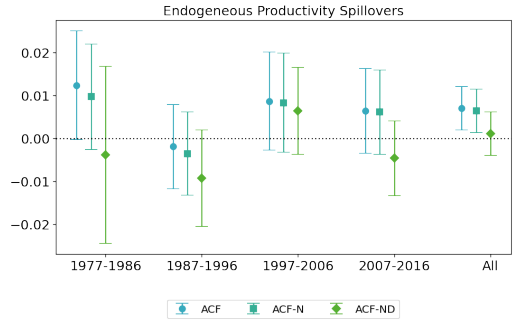


Figure 9: Spillover Estimates (Value-Added)
This figure shows point estimates and 95% confidence intervals of endogenous productivity spillovers from a value-added production function. See table 6 for standard errors.

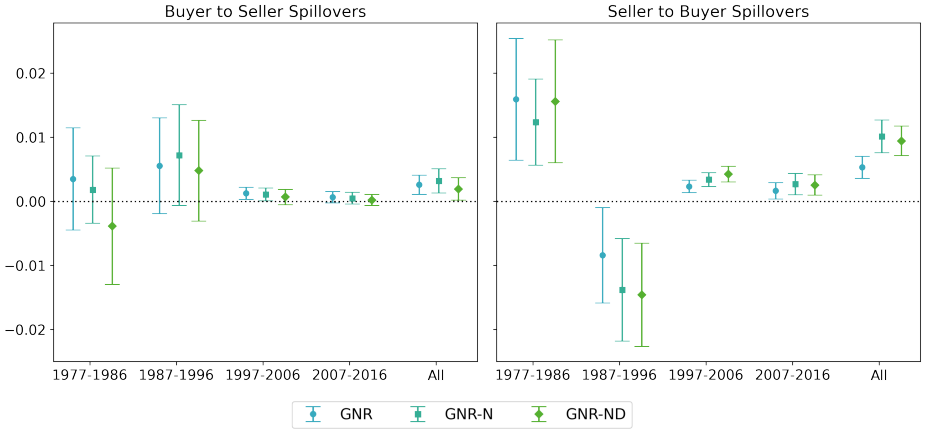


Figure 10: Spillover Estimates by Relationship Direction (Gross-Output)
This figure shows point estimates and 95% confidence intervals of endogenous productivity spillovers by direction of the relationship from a gross output production function. See table 7 for standard errors.



Figure 11: Spillover Estimates by Relationship Direction (Value-Added)
This figure shows point estimates and 95% confidence intervals of endogenous productivity spillovers by direction of the relationship from a value-added production function. See table 8 for standard errors.

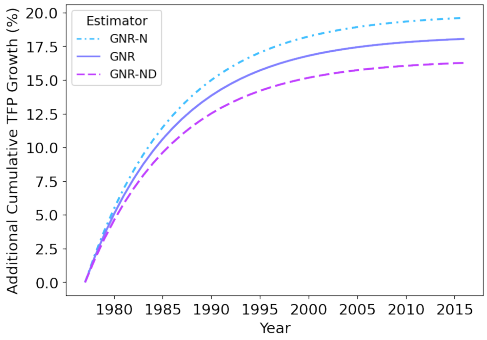


Figure 12: Cumulative Impact of Endogenous Productivity Spillovers over Time
This figure depicts a simulated path of log(TFP) for a firm with productivity equal to the 1978 average, assuming it is connected to the median firm in the same year. Endogenous productivity spillovers are assumed to be 0.0084, 0.009 and 0.0076 obtained from GNR, GNR-N and GNR-ND respectively. The bottom dotted line assumes that the firm experiences no spillovers.

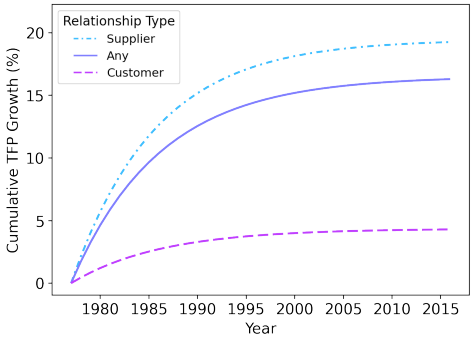


Figure 13: Cumulative Impact of Endogenous Productivity Spillovers by Relationship Type
This figure depicts a simulated path of log(TFP) for a firm with productivity equal to the 1978 average, assuming it is connected to the median firm in the same year. Endogenous productivity spillovers are assumed to be 0.002, 0.0095 and 0.0076 for customers, suppliers or either, respectively. The bottom dotted line assumes that the firm experiences no spillovers.

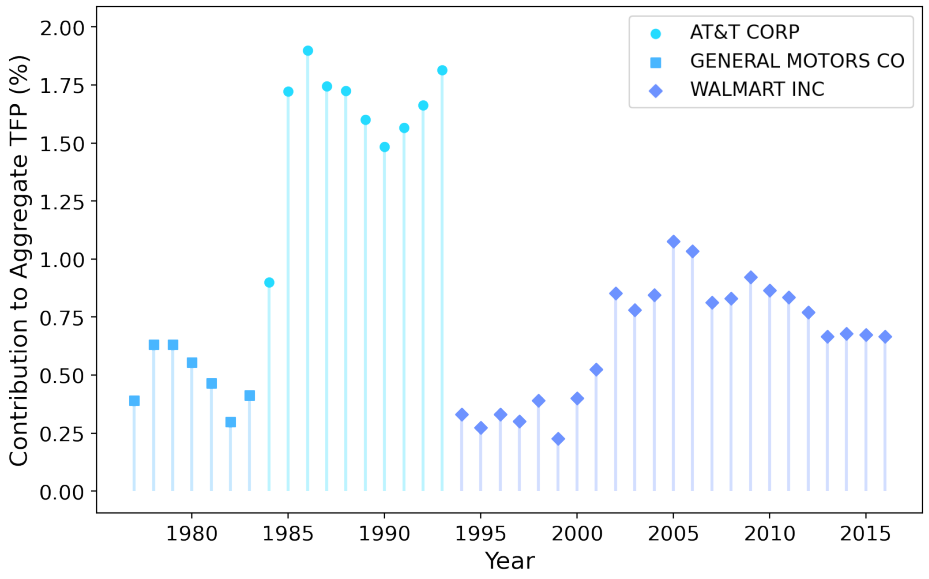
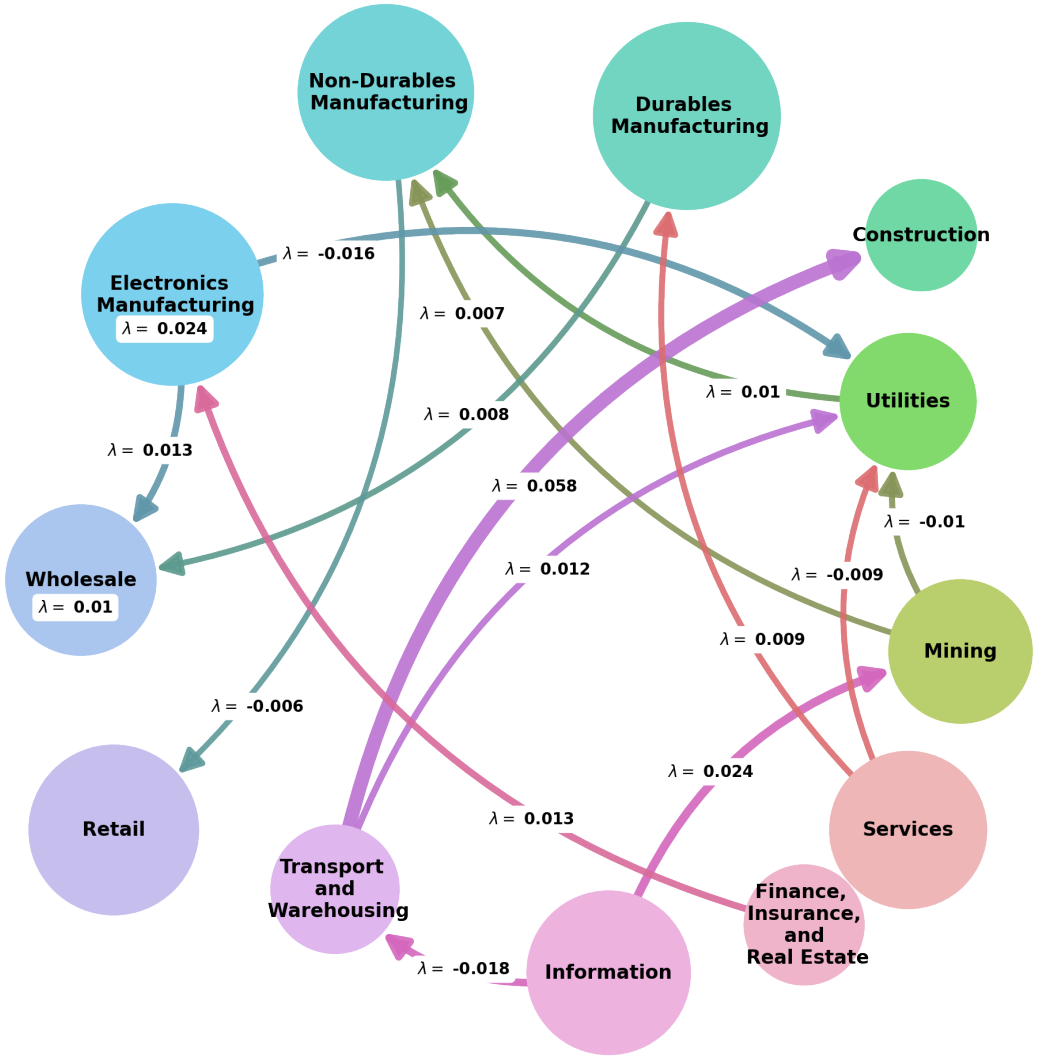


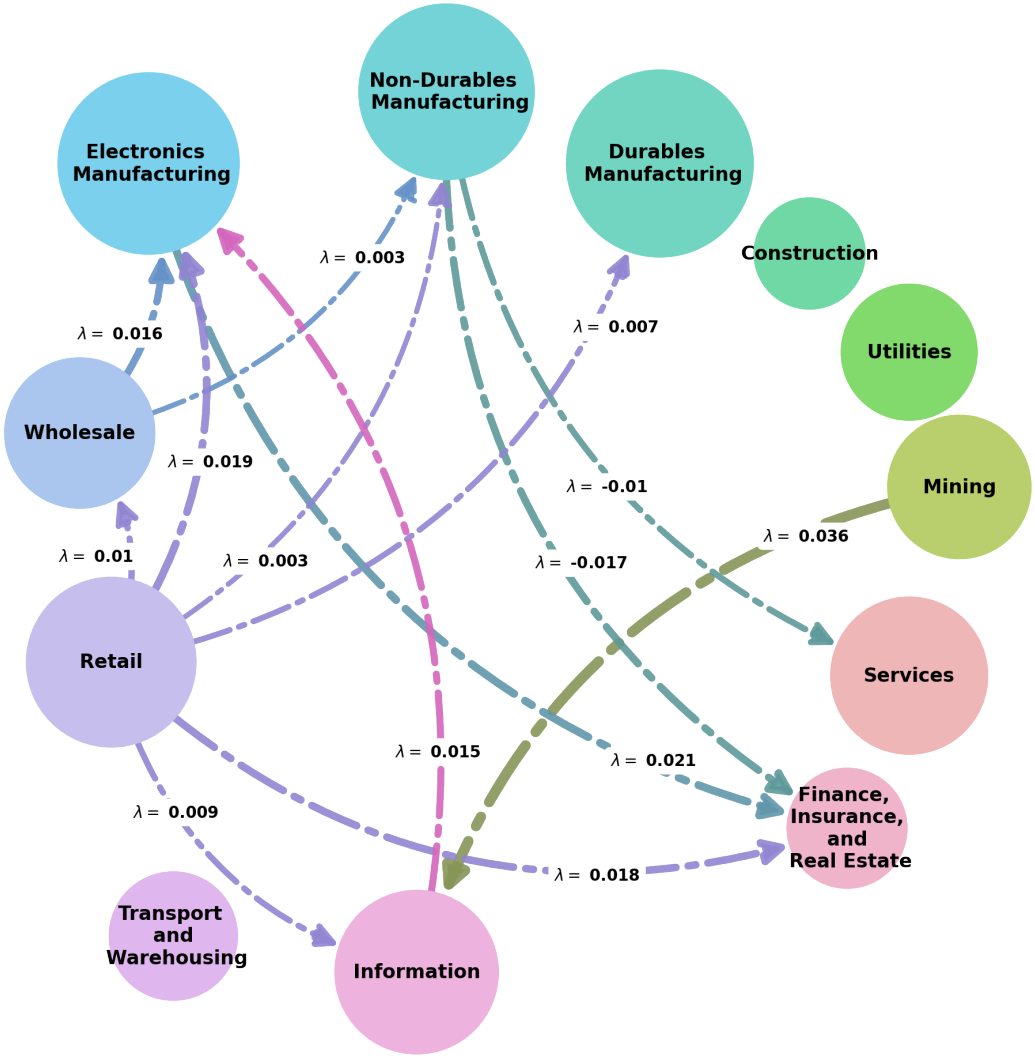
Figure 14: Contribution of Most Central Firms to Aggregate TFP
This figure shows the impact of a 10% increase in the TFP of the most central firm in each year to aggregate productivity through spillovers. The contribution of the most central firm j in year t is calculated as $\widehat{\lambda}\sum_{it} weight_{it}G_{ijt}$ where $\widehat{\lambda} = 0.02$ and $weight_{it} = \frac{Revenue_{it}}{Avg\ Revenue_t}$

Figure 15: Downstream Productivity Spillovers by Sector



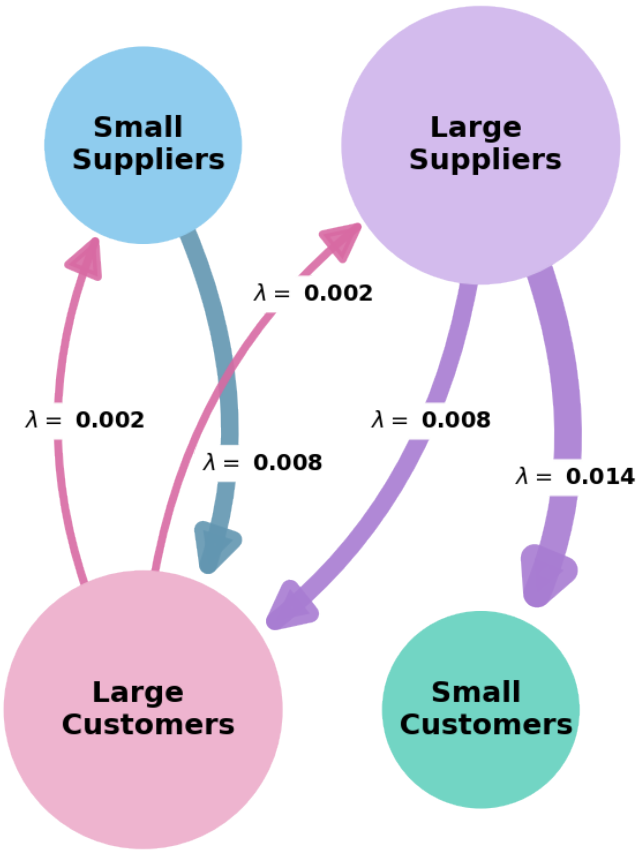
This figure shows downstream productivity spillovers (λ) that vary by the sector of the firm and its trading partners. Estimates are significant at the 5% level. Sector nodes are weighted by the total number of connections originating from or going to firms in the sector, across all time periods. See table 10 for the full set of coefficients.

Figure 16: Upstream Productivity Spillovers by Sector



This figure shows upstream productivity spillovers (λ) that vary by the sector of the firm and its trading partners. Estimates are significant at the 5% level. Industry nodes are weighted by the total number of connections originating from or going to firms in the sector, across all time periods. See table 10 for the full set of coefficients.

Figure 17: Endogenous Productivity Spillovers by Firm Size



This figure shows endogenous productivity spillovers that vary by firm size. Estimates are significant at the 10% level. See table 7 for the full set of coefficients.

Table 5: Endogenous Productivity Spillovers (Gross Output)

Dependent Variable: $\ln TFP_t$			
Period	Estimator	$\ln TFP_{t-1}$	Neighbors' $\ln TFP_t$
1977-1986	GNR	0.8401	0.0068
		(0.0205)	(0.0049)
	GNR-N	0.839	0.007
		(0.0207)	(0.0049)
	GNR-ND	0.8257	0.0001
		(0.0227)	(0.0057)
1987-1996	GNR	0.8313	-0.0077
		(0.0244)	(0.0043)
	GNR-N	0.8312	-0.0075
		(0.0244)	(0.0043)
	GNR-ND	0.8229	-0.0109
		(0.0271)	(0.0044)
1997-2006	GNR	0.858	0.0033
		(0.013)	(0.0042)
	GNR-N	0.8585	0.0041
		(0.0134)	(0.0042)
	GNR-ND	0.8586	0.0046
		(0.0147)	(0.0053)
2007-2016	GNR	0.8957	0.0145
		(0.0215)	(0.005)
	GNR-N	0.8963	0.0133
		(0.0214)	(0.0052)
	GNR-ND	0.8918	0.0109
		(0.023)	(0.0048)
All	GNR	0.9035	0.0084
		(0.0067)	(0.0023)
	GNR-N	0.9025	0.009
		(0.0067)	(0.0023)
	GNR-ND	0.8996	0.0076
		(0.0073)	(0.0024)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure in Lee (2003); Bramoullé et al. (2009). Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 6: Endogenous Productivity Spillovers (Value-Added)

Dependent Variable: $\ln TFP_t$			
Period	Estimator	$\ln TFP_{t-1}$	Neighbors' $\ln TFP_t$
1977-1986	ACF	0.8169	0.0123
		(0.0207)	(0.0065)
	ACF-N	0.8168	0.0097
		(0.0207)	(0.0062)
	ACF-ND	0.807	-0.0039
		(0.0214)	(0.0105)
1987-1996	ACF	0.8472	-0.002
		(0.0135)	(0.005)
	ACF-N	0.8479	-0.0036
		(0.0134)	(0.0049)
	ACF-ND	0.8482	-0.0093
		(0.0121)	(0.0057)
1997-2006	ACF	0.8679	0.0086
		(0.0116)	(0.0058)
	ACF-N	0.8682	0.0083
		(0.0116)	(0.0059)
	ACF-ND	0.8685	0.0064
		(0.0104)	(0.0051)
2007-2016	ACF	0.8774	0.0063
		(0.0321)	(0.005)
	ACF-N	0.8776	0.0061
		(0.032)	(0.005)
	ACF-ND	0.8691	-0.0046
		(0.0358)	(0.0044)
All	ACF	0.8687	0.007
		(0.0095)	(0.0026)
	ACF-N	0.8688	0.0064
		(0.0095)	(0.0026)
	ACF-ND	0.8663	0.001
		(0.0101)	(0.0026)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a value-added production function (in logs) estimated with the standard Ackerberg et al. (2015) procedure (ACF), or with modifications to accommodate network effects (ACF-N) and network differencing (ACF-ND). Network effects for ACF are estimated using the generalized 2SLS procedure in Lee (2003); Bramoullé et al. (2009). Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 7: Productivity Spillovers by Relationship Direction (Gross Output)

Dependent Variable: $\ln TFP_t$			
Period	Estimator	Customers' $\ln TFP_t$	Suppliers' $\ln TFP_t$
1977-1986	GNR	0.0035	0.0159
		(0.0041)	(0.0048)
	GNR-N	0.0019	0.0124
		(0.0027)	(0.0034)
	GNR-ND	-0.0038	0.0156
		(0.0046)	(0.0049)
1987-1996	GNR	0.0056	-0.0084
		(0.0038)	(0.0038)
	GNR-N	0.0072	-0.0138
		(0.004)	(0.0041)
	GNR-ND	0.0048	-0.0145
		(0.004)	(0.0041)
1997-2006	GNR	0.0013	0.0023
		(0.0005)	(0.0005)
	GNR-N	0.0011	0.0034
		(0.0005)	(0.0006)
	GNR-ND	0.0007	0.0043
		(0.0006)	(0.0006)
2007-2016	GNR	0.0007	0.0017
		(0.0004)	(0.0006)
	GNR-N	0.0005	0.0027
		(0.0005)	(0.0008)
	GNR-ND	0.0002	0.0026
		(0.0004)	(0.0008)
All	GNR	0.0026	0.0053
		(0.0008)	(0.0009)
	GNR-N	0.0032	0.0102
		(0.001)	(0.0013)
	GNR-ND	0.002	0.0095
		(0.0009)	(0.0012)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 8: Productivity Spillovers by Relationship Direction (Value-Added)

Dependent Variable: $\ln TFP_t$			
Period	Estimator	Customers' $\ln TFP_t$	Suppliers' $\ln TFP_t$
1977-1986	ACF	0.0018	0.0024
		(0.0014)	(0.0014)
	ACF-N	0.0015	0.0027
		(0.0014)	(0.0014)
	ACF-ND	-0.0003	0.002
		(0.0016)	(0.0015)
1987-1996	ACF	0.0011	0.0012
		(0.0008)	(0.0008)
	ACF-N	0.0011	0.0007
		(0.0008)	(0.0008)
	ACF-ND	0.0011	0.0004
		(0.0008)	(0.0008)
1997-2006	ACF	0.0013	0.0016
		(0.0006)	(0.0006)
	ACF-N	0.0012	0.002
		(0.0006)	(0.0006)
	ACF-ND	0.0007	0.0022
		(0.0006)	(0.0006)
2007-2016	ACF	0.001	0.0012
		(0.0005)	(0.0005)
	ACF-N	0.001	0.0014
		(0.0004)	(0.0005)
	ACF-ND	0.0011	0.001
		(0.0004)	(0.0004)
All	ACF	0.0013	0.0016
		(0.0003)	(0.0004)
	ACF-N	0.0012	0.0018
		(0.0003)	(0.0004)
	ACF-ND	0.0011	0.0016
		(0.0003)	(0.0003)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a value-added production function (in logs) estimated with the standard Ackerberg et al. (2015) procedure (ACF), or with modifications to accommodate network effects (ACF-N) and network differencing (ACF-ND). Network effects for ACF are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 9: Productivity Spillovers by Relationship Dynamics (Gross Output)

		Dependent Variable: $\ln TFP_t$			
Period	Estimator	Continuing Customers' $\ln TFP_t$	New Customers' $\ln TFP_t$	Continuing Suppliers' $\ln TFP_t$	New Suppliers' $\ln TFP_t$
1977-1986	GNR	0.0032	0.0006	0.0143	0.0073
		(0.0033)	(0.0039)	(0.0037)	(0.0038)
	GNR-N	0.0	0.0001	0.0087	0.0052
		(0.0017)	(0.0018)	(0.002)	(0.0018)
	GNR-ND	-0.0004	-0.0019	0.0113	0.0049
		(0.0023)	(0.0022)	(0.0024)	(0.0023)
1987-1996	GNR	0.0072	0.0033	-0.0035	-0.0118
		(0.0035)	(0.0039)	(0.0041)	(0.0035)
	GNR-N	0.0074	0.0082	-0.009	-0.0194
		(0.0041)	(0.0049)	(0.0049)	(0.004)
	GNR-ND	-0.0128	0.0101	-0.0009	-0.0175
		(0.0046)	(0.0064)	(0.0101)	(0.0081)
1997-2006	GNR	-0.0007	0.0005	0.0003	0.0013
		(0.0004)	(0.0004)	(0.0003)	(0.0003)
	GNR-N	-0.0009	0.0003	0.0014	0.0025
		(0.0004)	(0.0004)	(0.0004)	(0.0004)
	GNR-ND	-0.0011	0.0002	0.0019	0.0031
		(0.0005)	(0.0005)	(0.0005)	(0.0004)
2007-2016	GNR	0.0005	0.0006	0.0015	0.0013
		(0.0004)	(0.0004)	(0.0005)	(0.0004)
	GNR-N	0.0005	0.0004	0.0031	0.0026
		(0.0005)	(0.0005)	(0.0008)	(0.0006)
	GNR-ND	0.0003	0.0003	0.0031	0.0027
		(0.0005)	(0.0005)	(0.0009)	(0.0007)
All	GNR	0.0008	0.0017	0.0028	0.0038
		(0.0006)	(0.0006)	(0.0006)	(0.0006)
	GNR-N	0.001	0.0017	0.0067	0.0077
		(0.0007)	(0.0007)	(0.0009)	(0.0008)
	GNR-ND	0.0005	0.0013	0.0071	0.0082
		(0.0007)	(0.0008)	(0.001)	(0.0009)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 10: Productivity Spillovers by Sector (Gross Output)

	Dependent Variable: $\ln TFP_t$ Firm's Sector											
Partners' Sector	Mining	Utilities	Constr	Durables	Non-Durables	Electronics	Wholesale	Retail	Trans & WH	Info	FIRE	Services
Mining	-0.0084	-0.0096	-0.0064	0.0017	0.0075	-0.0161	0.0172	0.0094	0.0102	0.0357	0.0042	0.0028
	(0.0064)	(0.0038)	(0.0331)	(0.005)	(0.0027)	(0.0326)	(0.0104)	(0.008)	(0.0092)	(0.0111)	(0.0085)	(0.008)
Utilities	-0.0032	-0.0007	0.0061	0.003	0.01	0.0134	0.0041	-0.0028	0.0018	-0.0024	-0.0038	0.0105
	(0.0059)	(0.0028)	(0.0104)	(0.0027)	(0.0021)	(0.0082)	(0.0083)	(0.0084)	(0.0065)	(0.0125)	(0.0178)	(0.006)
Construction	-0.0192	0.0014	0.0147	0.0055	-0.0058	-0.0079	0.0113	-0.0075	0.017	0.0043	-0.0005	0.0282
	(0.0312)	(0.0041)	(0.0115)	(0.0059)	(0.0036)	(0.0133)	(0.0076)	(0.009)	(0.0228)	(0.0035)	(0.0061)	(0.0218)
Durables Mfg	0.0077	0.0002	0.0265	0.0025	0.0012	-0.0036	0.0079	-0.0016	0.0029	0.0039	0.0052	0.0032
	(0.0065)	(0.003)	(0.0231)	(0.0022)	(0.0016)	(0.0024)	(0.0027)	(0.0016)	(0.0035)	(0.0034)	(0.0063)	(0.0034)
Non-Durables Mfg	-0.0061	-0.0014	-0.0028	0.0001	-0.001	-0.0067	0.0034	-0.0062	-0.0065	-0.0026	-0.0175	-0.0102
	(0.0052)	(0.0028)	(0.0132)	(0.0018)	(0.0013)	(0.0048)	(0.0027)	(0.0016)	(0.0038)	(0.0044)	(0.0085)	(0.0038)
Electronics Mfg	-0.0433	-0.0157	-0.0275	-0.0003	-0.0043	0.0244	0.013	0.0021	0.0079	0.0007	0.0207	0.0001
	(0.0501)	(0.0055)	(0.0355)	(0.0046)	(0.0038)	(0.0029)	(0.0031)	(0.0032)	(0.0058)	(0.0031)	(0.0091)	(0.0043)
Wholesale	-0.0052	0.0126	0.0121	0.0022	0.0032	0.0165	0.01	0.0017	0.0033	0.0022	-0.0132	0.0027
	(0.0118)	(0.0075)	(0.0164)	(0.0021)	(0.0012)	(0.0021)	(0.0038)	(0.0014)	(0.0104)	(0.0024)	(0.0118)	(0.004)
Retail	0.0113	-0.0037	-0.0075	0.0074	0.003	0.0194	0.0104	0.0012	-0.0013	0.009	0.018	0.0017
	(0.0118)	(0.0104)	(0.016)	(0.0028)	(0.0013)	(0.0036)	(0.0019)	(0.0026)	(0.004)	(0.0041)	(0.0046)	(0.0044)
Transport and Warehousing	0.0079	0.0116	0.0575	0.0037	0.0033	0.0067	0.0078	0.0022	-0.0031	0.0007	-0.0088	0.0
	(0.0094)	(0.0034)	(0.0184)	(0.0041)	(0.002)	(0.0059)	(0.0109)	(0.0039)	(0.0042)	(0.0081)	(0.0073)	(0.0103)
Information	0.0244	-0.0023	0.0085	-0.0035	-0.001	0.0151	0.0064	-0.001	-0.018	0.0041	0.0066	-0.0074
	(0.0118)	(0.0069)	(0.017)	(0.0039)	(0.0028)	(0.0025)	(0.0049)	(0.0026)	(0.0048)	(0.0028)	(0.0047)	(0.004)
Finance, Insur & Real Estate	0.0004	0.005	-0.0134	0.0	-0.0044	0.0134	-0.0026	0.0017	0.0045	0.0042	-0.0021	0.0048
	(0.0112)	(0.0143)	(0.0128)	(0.0044)	(0.0025)	(0.0034)	(0.0149)	(0.002)	(0.0043)	(0.0031)	(0.0043)	(0.0036)
Services	0.0035	-0.0088	0.0083	0.0088	0.0042	0.0022	0.0084	0.0042	0.0025	0.0052	0.009	0.0025
	(0.0117)	(0.0036)	(0.0253)	(0.0023)	(0.0023)	(0.0028)	(0.0072)	(0.0026)	(0.005)	(0.0031)	(0.0048)	(0.0037)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Sectors are determined according to the BEA industry classification. Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 11: Productivity Spillovers by Firm Size & Relationship Direction (Gross Output)

Partner Size	Relationship	Firm Size	Dependent Variable: $\ln TFP_t$ 1977-1986	1987-1996	1997-2006	2007-2016	All
Large	Customers	Large	0.0012	0.0033	-0.0001	0.0003	0.002
			(0.0031)	(0.0073)	(0.0004)	(0.0004)	(0.0008)
		Small	-0.0032	-0.0028	0.0005	-0.0004	0.0021
			(0.0037)	(0.0056)	(0.0006)	(0.0006)	(0.001)
	Suppliers	Large	0.0141	-0.0226	0.0024	0.0026	0.0083
			(0.0032)	(0.0103)	(0.0004)	(0.0007)	(0.001)
		Small	0.0509	-0.0638	0.0046	-0.005	0.0143
			(0.0273)	(0.0208)	(0.0024)	(0.0025)	(0.0077)
Small	Customers	Large	-0.0115	-0.0701	-0.0058	-0.0059	-0.0091
			(0.0106)	(0.0447)	(0.0026)	(0.0064)	(0.0076)
		Small	-0.0004	-0.0387	0.0	-0.0008	-0.0045
			(0.0127)	(0.0329)	(0.0021)	(0.0027)	(0.0052)
	Suppliers	Large	0.0122	-0.0124	0.0021	0.0023	0.0081
			(0.0034)	(0.0054)	(0.0004)	(0.0006)	(0.0009)
		Small	0.0502	-0.0144	0.0032	0.0	0.0075
			(0.0227)	(0.0133)	(0.0019)	(0.003)	(0.0053)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for ACF are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Large firms are businesses with 500 or more employees.Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 12: Productivity Spillovers by Varying Firm Size Cutoffs (Gross Output)

Dependent Variable: $\ln TFP_t$						
			Firm's Sector			
Partner Size	Relationship	Firm Size	500	1000	5000	Median
Large	Customers	Large	0.002	0.0017	0.0015	0.0011
			(0.0008)	(0.0008)	(0.0008)	(0.0008)
		Small	0.0021	0.0036	0.0019	0.0015
			(0.001)	(0.0011)	(0.001)	(0.001)
	Suppliers	Large	0.0083	0.008	0.006	0.0088
			(0.001)	(0.001)	(0.0008)	(0.001)
		Small	0.0143	0.0094	0.0086	0.0093
			(0.0077)	(0.0036)	(0.0027)	(0.0016)
Small	Customers	Large	-0.0091	-0.001	-0.0038	0.0019
			(0.0076)	(0.0061)	(0.0039)	(0.0015)
		Small	-0.0045	-0.0051	0.0007	0.0026
			(0.0052)	(0.0036)	(0.0012)	(0.0012)
	Suppliers	Large	0.0081	0.0086	0.008	0.0074
			(0.0009)	(0.001)	(0.0012)	(0.0011)
		Small	0.0075	0.0074	0.0091	0.0088
			(0.0053)	(0.0033)	(0.0013)	(0.0013)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Large firms are defined by having at least as many employees as the cutoffs indicated above. The median cutoff is determined by industry and year. Standard errors are in parentheses. All specifications include industry and year fixed effects.

9 Concluding Remarks

This paper examines how efficiency gains are transmitted through vertical relationships. The existence of spillovers implies a form of firm interdependence that matters for consistent estimation of production functions. Using Monte Carlo experiments, I show that endogenous spillovers—the effect of the average productivity of a firm's neighbors on its own productivity—are an important source of bias when not accounted for in TFP estimation. Furthermore, the direction of this bias cannot always be clearly predicted *a priori*, and varies by the density of the network and the persistence of productivity. However, for moderately sparse networks, standard approaches may deliver reasonably unbiased estimates of production function elasticities and spillovers.

Under additional assumptions on firms' information sets and the structure of the network, I propose a methodology that can flexibly accommodate various network effects and endogenous network formation, and can be applied to both gross output and value-added production functions. I show experimentally that it performs better than standard approaches as long as the network is not too dense and productivity is sufficiently persistent.

Using data from *Compustat* on supplier-customer relationships in the US, I investigate the extent of productivity spillovers in from 1977 to 2016. I find that firms benefit from having more productive buyers and sellers, with both large and small suppliers having a larger effect than customers. Furthermore, the cumulative impact of spillovers over the 4 decades in the sample could mean a 16 percent difference in efficiency when compared to a no-spillover scenario.

Estimates suggest that if the most connected firm in a given year was 10 percent more productive, spillovers would lead to an increase in aggregate TFP of 0.2 to 1.9 percent. This also works in the opposite direction: a significant decline in productivity of central firms could mean substantial second-order impacts to US aggregate efficiency due to the interdependence of firms' activities through supply chains. This suggests that industrial and trade policies that could potentially affect the productivity of well-connected firms needs to account for potential indirect effects both upstream and downstream.

The sectoral composition of the production network plays a large role in the size and transmission of productivity gains. I find substantial heterogeneity in the size and

spillovers between and within sectors, with electronics manufacturers benefiting from efficiency gains from most sectors, while retailers and services boost other sectors.

Consistent with my Monte Carlo experiments, estimates from standard approaches empirically yielded estimates of network effects that were similar to those obtained from my procedure, because the observed network density fell within the region where bias in spillover estimates was minimized. While this is reassuring for studies conducted on networks with similar levels of sparsity, caution should be taken when networks are much sparser or denser.

Results differed between gross output and value-added specifications. As discussed in Gandhi et al. (2017), value-added and gross output productivity measures may vary significantly and lead to substantively different policy implications about the dispersion of firm productivity. My paper reveals that the choice of production function also matters for the estimation of productivity spillovers.

References

Ackerberg, D. A., K. Caves, and G. Frazer (2015). Identification properties of recent production function estimators. *Econometrica 83*(6), 2411–2451. (Cited on pages 4, 14, 26, 39, 49, 50, iii, ix, xvi, xvii, xxi, xxii, xxiii, and xxiv.)

Alfaro-Urena, A., I. Manelici, and J. P. Vasquez (2022). The effects of joining multinational supply chains: New evidence from firm-to-firm linkages. *The Quarterly Journal of Economics 137*(3), 1495–1552. (Cited on page 3.)

Alvarez, R. and R. A. López (2008). Is exporting a source of productivity spillovers? *Review of World Economics 144*(4), 723–749. (Cited on page 3.)

Amiti, M., C. Duprez, J. Konings, and J. Van Reenen (2023). Fdi and superstar spillovers: Evidence from firm-to-firm transactions. Technical report, National Bureau of Economic Research. (Cited on page 3.)

Arduini, T., E. Patacchini, E. Rainone, et al. (2015). Parametric and semiparametric IV estimation of network models with selectivity. Technical report, Einaudi Institute for Economics and Finance (EIEF). (Cited on pages 27 and 30.)

Atalay, E., A. Hortaçsu, J. Roberts, and C. Syverson (2011). Network structure of production. *Proceedings of the National Academy of Sciences 108*(13), 5199–5202. (Cited on page 7.)

Bazzi, S., A. V. Chari, S. Nataraj, and A. D. Rothenberg (2017). Identifying productivity spillovers using the structure of production networks. (Cited on page 4.)

Blundell, R. and S. Bond (2000). GMM estimation with persistent panel data: an application to production functions. *Econometric reviews 19*(3), 321–340. (Cited on pages 19 and i.)

Bramoullé, Y., H. Djebbari, and B. Fortin (2009). Identification of peer effects through social networks. *Journal of Econometrics 150*, 41–55. (Cited on pages 5, 17, 23, 25, 37, 49, and xxvi.)

Collard-Wexler, A. and J. De Loecker (2016). Production function estimation with measurement error in inputs. Technical report, National Bureau of Economic Research. (Cited on pages iii, v, ix, and x.)

De Loecker, J. (2013). Detecting learning by exporting. *American Economic Journal: Microeconomics 5*(3), 1–21. (Cited on pages 4 and 18.)

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2016). Prices, markups, and trade reform. *Econometrica 84*(2), 445–510. (Cited on page 4.)

De Loecker, J. and C. Syverson (2021). An industrial organization perspective on productivity. In *Handbook of Industrial Organization*, Volume 4, pp. 141–223. Elsevier. (Cited on page 3.)

Dieye, R. and B. Fortin (2017). Gender peer effects heterogeneity in obesity. (Cited on pages 5, 36, 50, 51, 52, 53, 54, xxi, xxii, xxiii, xxiv, xxvi, xxvii, xxviii, xxix, and xxx.)

Erdős, P. and A. Rényi (1960). On the evolution of random graphs. *Publications of the Mathematical Institute of the Hungarian Academy of Sciences 5*(1), 17–60. (Cited on pages iv and ix.)

Gandhi, A., S. Navarro, and D. Rivers (2017). How heterogeneous is productivity? a comparison of gross output and value added. (Cited on page 56.)

Gandhi, A., S. Navarro, and D. A. Rivers (2020). On the identification of gross output production functions. *Journal of Political Economy 128*(8), 2973–3016. (Cited on pages 4, 14, 18, 32, 39, 49, 50, 51, 52, 53, 54, xxvi, xxvii, xxviii, xxix, and xxx.)

Garcia-Marin, A. and N. Voigtländer (2019). Exporting and plant-level efficiency gains: It's in the measure. *Journal of Political Economy 127*(4), 1777–1825. (Cited on page 4.)

Goldsmith-Pinkham, P. and G. W. Imbens (2013). Social networks and the identification of peer effects. *Journal of Business & Economic Statistics 31*(3), 253–264. (Cited on pages 17, 27, and 29.)

Graham, B. S. (2017). An econometric model of network formation with degree heterogeneity. *Econometrica 85*(4), 1033–1063. (Cited on page 28.)

Hsieh, C.-S. and L. F. Lee (2016). A social interactions model with endogenous friendship formation and selectivity. *Journal of Applied Econometrics 31*(2), 301–319. (Cited on page 27.)

İmrohoroğlu, A. and Ş. Tüzel (2014). Firm-level productivity, risk, and return. *Management Science 60*(8), 2073–2090. (Cited on page xviii.)

Javorcik, B. S. (2004). Does foreign direct investment increase the productivity of domestic firms? in search of spillovers through backward linkages. *American Economic Review 94*(3), 605–627. (Cited on pages 3 and 17.)

Keller, W. (2010). International trade, foreign direct investment, and technology spillovers. *Handbook of the Economics of Innovation 2*, 793–829. (Cited on pages 3 and 5.)

Keller, W. and S. R. Yeaple (2009). Multinational enterprises, international trade, and productivity growth: firm-level evidence from the united states. *The Review of Economics and Statistics 91*(4), 821–831. (Cited on page 3.)

Lee, L. F. (2003). Best spatial two-stage least squares estimators for a spatial autoregressive model with autoregressive disturbances. *Econometric Reviews 22*(4), 307–335. (Cited on pages 4, 17, 23, 37, 49, and xxvi.)

Lee, L. F. and J. Yu (2016). Identification of spatial Durbin panel models. *Journal of Applied Econometrics 31*(1), 133–162. (Cited on pages 5 and 17.)

Levinsohn, J. and A. Petrin (2003). Estimating production functions using inputs to control for unobservables. *The Review of Economic Studies 70*(2), 317–341. (Cited on page 4.)

Lim, K. et al. (2017). Firm-to-firm trade in sticky production networks. In *2017 Meeting Papers*, Number 280. Society for Economic Dynamics. (Cited on page 7.)

Liu, X. and L. F. Lee (2010). GMM estimation of social interaction models with centrality. *Journal of Econometrics 159*(1), 99–115. (Cited on pages 35 and 36.)

Liu, X., E. Patacchini, and Y. Zenou (2014). Endogenous peer effects: local aggregate or local average? *Journal of Economic Behavior & Organization 103*, 39–59. (Cited on pages 35 and 36.)

Malikov, E. and S. Zhao (2021, 08). On the Estimation of Cross-Firm Productivity Spillovers with an Application to FDI. *The Review of Economics and Statistics*, 1–46. (Cited on page 5.)

Manski, C. F. (1993). Identification of endogenous social effects: The reflection problem. *The Review of Economic Studies 60*(3), 531–542. (Cited on page 16.)

Merlevede, B. and A. Theodorakopoulos (2023). Intangibles within firm boundaries. (Cited on page 5.)

Olley, G. S. and A. Pakes (1996). The dynamics of productivity in the telecommunications equipment. *Econometrica 64*(6), 1263–1297. (Cited on pages 4, 27, and 30.)

Patacchini, E., E. Rainone, and Y. Zenou (2017). Heterogeneous peer effects in education. *Journal of Economic Behavior & Organization 134*, 190–227. (Cited on pages 5, 36, 50, 51, 52, 53, 54, xxi, xxii, xxiii, xxiv, xxvi, xxvii, xxviii, xxix, and xxx.)

Qu, X., L. F. Lee, and J. Yu (2017). QML estimation of spatial dynamic panel data models with endogenous time varying spatial weights matrices. *Journal of Econometrics 197*(2), 173–201. (Cited on page 27.)

Serpa, J. C. and H. Krishnan (2018). The impact of supply chains on firm-level productivity. *Management Science 64*(2), 511–532. (Cited on pages 3, 7, 17, and 27.)

Syverson, C. (2011). What determines productivity? *Journal of Economic literature 49*(2), 326–65. (Cited on page 3.)

Van Biesebroeck, J. (2007). Robustness of productivity estimates. *The Journal of Industrial Economics 55*(3), 529–569. (Cited on pages iii, v, ix, and x.)

Wooldridge, J. M. (2009). On estimating firm-level production functions using proxy variables to control for unobservables. *Economics Letters 104*(3), 112–114. (Cited on page 4.)

Appendices

A Derivation of Bias Terms

In this section, I derive expressions for the bias in production function elasticities shown in section 3.3.

$$y_t = \alpha_\ell \ell_t + \alpha_k k_t + \omega_t + \varepsilon_t$$

$$\omega_t = \rho(I - \lambda G_t)^{-1}\omega_{t-1} + (I - \lambda G_t)^{-1}\zeta_{it} = \rho \sum_{s=0} \lambda^s G_t^s \omega_{t-1} + \sum_{s=0} \lambda^s G_t^s \zeta_t$$

$$\implies y_t = \alpha_\ell \ell_t + \alpha_k k_t + \rho \sum_{s=0} \lambda^s G_t^s \omega_{t-1} + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \varepsilon_t$$

$$\omega_{t-1} = \varphi_{t-1} - \alpha_\ell \ell_{t-1} - \alpha_k k_{it-1}$$

$$\implies y_t = \alpha_\ell \ell_t + \alpha_k k_t + \rho \sum_{s=0} \lambda^s G_t^s \left(\varphi_{t-1} - \alpha_\ell \ell_{t-1} - \alpha_k k_{it-1}\right) + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_{it} + \varepsilon_t$$

$$y_{t-1} = \varphi_{t-1} + \varepsilon_t$$

$$\implies y_t = \alpha_\ell \ell_t + \alpha_k k_t + \rho \sum_{s=0}^{\infty} \lambda^s G_t^s \left(y_{t-1} - \alpha_\ell \ell_{t-1} - \alpha_k k_{t-1} - u_{t-1}\right) + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \varepsilon_t$$

Let $\Delta^G x_t = x_t - \rho \sum_{s=0}^{\infty} \lambda^s G_t^s x_{t-1}$, $\Delta_{x_t}^{err} = \rho \sum_{s=1}^{\infty} \lambda^s G_t^s x_{t-1}$ and $\Delta x_t = x_t - \rho x_{t-1} = \Delta^G x_t + \Delta_{x_t}^{err}$. This implies:

$$\Delta^G y_t = \alpha_\ell \Delta^G \ell_t + \alpha_k \Delta^G k_t + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \Delta^G \varepsilon_t \tag{57}$$

This is equivalent to the dynamic panel approach in Blundell and Bond (2000). However, growth in output, labor and capital have been purged of the variation from network effects in the previous period. When we assume no spillovers, we estimate:

$$\Delta y_t = \alpha_\ell \Delta \ell_t + \alpha_k \Delta k_t + u_t \tag{58}$$

Therefore, in the linear AR1 case, ignoring spillovers is equivalent to introducing non-classical measurement error into both output and inputs.

Bias from ignoring spillovers can also be characterized as an omitted variables

problem. By estimating equation (58), where $u_t = \rho \sum_{s=1}^{\infty} \lambda^s G_t^s \omega_{t-1} + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \varepsilon_t$. That is, the standard ACF procedure succeeds in eliminating the endogeneity problem that arises from input decisions depending on its own productivity, but is unable to account for the influence of its network's past productivity.

In either case, an instrumental variable approach would help to eliminate the problem. The key would be to find variables that are correlated with changes to labor and capital but uncorrelated with output, particularly the input choices and output of other firms.

In the OP case where the labor elasticity is estimated in the first stage, the second stage is equivalent to estimating:

$$\Delta^G \widetilde{y}_t = \alpha_k \Delta^G k_t + \sum_{s=0}^{\infty} \lambda^s G_t^s \zeta_t + \Delta^G \varepsilon_t \tag{59}$$

$$\tag{60}$$

where $\widetilde{y}_t = y_t - \widehat{\alpha}_\ell \ell_t$. Then by estimating $\Delta\widetilde{y}_t = \alpha_k \Delta k_t + u_t$ under the standard assumption of no-spillovers:

$$plim\ \widehat{\alpha}_k = \frac{cov(\Delta k_t, \Delta\widetilde{y}_t)}{var(\Delta k_t)} \tag{61}$$

$$plim\ = \alpha_k \left(1 - \rho \sum_{s=1}^{\infty} \lambda^s \frac{cov(\Delta k_t, G_t^s k_{t-1})}{var(\Delta k_t)}\right) + \rho \sum_{s=1}^{\infty} \lambda^s \frac{cov(\Delta k_t, G_t^s \widetilde{y}_{t-1})}{var(\Delta k_t)} \tag{62}$$

When productivity is mismeasured by ignoring spillovers, the resulting estimates also result in incorrect conclusions about spillover effects. When (α_ℓ, α_k) are consistently estimated, and

$$\widehat{\omega}_t = \widehat{\varphi}_t - \widehat{\alpha}_\ell \ell_t - \widehat{\alpha}_k k_t \tag{63}$$

$$plim\ \widehat{\omega}_t = \varphi_t - \alpha_\ell \ell_t - \alpha_k k_t = \omega_t \tag{64}$$

However, when we estimate $(\widetilde{\alpha}_\ell, \widetilde{\alpha}_k) = (\widehat{\alpha}_\ell + \alpha_\ell^{err}, \widehat{\alpha}_k + \alpha_k^{err})$, to obtain $\widetilde{\omega}_t = \widehat{\varphi}_t - \widetilde{\alpha}_\ell \ell_t - \widetilde{\alpha}_k k_t$. Then

$$\widetilde{\omega}_t = \widehat{\varphi}_t - \widetilde{\alpha}_\ell \ell_t - \widetilde{\alpha}_k k_t = \widehat{\varphi}_t - \widehat{\alpha}_\ell \ell_t - \widehat{\alpha}_k k_t - (\alpha_\ell^{err} \ell_t + \alpha_k^{err} k_t) = \widehat{\omega}_t - \omega_t^{err} \tag{65}$$

where $\omega_t^{err} = \alpha_\ell^{err} \ell_t + \alpha_k^{err} k_t$ In the generalized 2SLS procedure for estimating network

effects, we estimate $\widetilde{\lambda}$ in the first stage by using $G_t\widetilde{\omega}_{t-1}$ as an instrument for $G_t\widetilde{\omega}_t$ in this equation:[31] The true model is:

$$\omega_t = \rho\omega_{t-1} + \lambda G_t\omega_t + \zeta_t$$

but we estimate:

$$\widetilde{\omega}_t = \rho\widetilde{\omega}_{t-1} + \lambda G_t\widetilde{\omega}_t + v_t$$

B Monte Carlo Setup

The Monte Carlo setup closely follows Collard-Wexler and De Loecker (2016), Van Biesebroeck (2007) and Ackerberg et al. (2015) with modifications for network generation and the inclusion of spillovers in the productivity process. I generate a balanced panel of 1000 firms over 10 time periods.

B.1 Production Function

I use a structural value-added production function that is Leontief in materials.

$$Y_{it} = \min\{L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}}, \alpha_m M_{it}\} e^{\varepsilon_{it}} \tag{66}$$

$$\implies Y_{it} = L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}+\varepsilon_{it}} = \alpha_m M_{it} e^{\varepsilon_{it}} \tag{67}$$

$$\text{In logs, } y_{it} = \alpha_\ell \ell_{it} + \alpha_k k_{it} + \omega_{it} + \varepsilon_{it} \tag{68}$$

where $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. I set $\alpha_\ell = 0.6, \alpha_k = 0.4$ and $\sigma_\varepsilon^2 = 1$

B.2 Productivity Process and Network

Productivity evolves according to an AR1 process that allows for contemporaneous endogenous productivity spillovers. For ease of notation, I write the equation in vectorized form:

$$\omega_t = \beta_1\iota + \rho\omega_{t-1} + \lambda G_t\omega_t + \zeta_t \tag{69}$$

[31] Further lags of the network effect can be used ($G_t^2\widetilde{\omega}_t, G_t^3\widetilde{\omega}_t$ and so on). However, for ease of exposition, I focus on the just-identified case.

where $\zeta_{it} \sim \mathcal{N}(0, \sigma_\zeta^2)$. I set $\sigma_\zeta^2 = 5$. I generate productivity using the reduced form of the above equation:

$$\omega_t = (\mathrm{I} - \lambda G_t)^{-1} (\beta_1 \iota + \rho \omega_{t-1} + \zeta_t) \tag{70}$$

G_t is the interaction matrix defined as in section 3.2 derived from the network. I generate exogenous networks using Erdős and Rényi (1960) graphs, also known as binomial graphs. Firms are edges are formed $A_{ijt} \overset{i.i.d.}{\sim} \text{Bern}(p)$.

B.3 Intermediate Input Demand

$$M_{it} = \frac{1}{\alpha_m} K_{it}^{\alpha_k} L_{it}^{\alpha_\ell} e^{\omega_{it}} \tag{71}$$

$$\text{In logs, } m_{it} = \alpha_k k_{it} + \alpha_\ell \ell_{it} + \omega_{it} - \ln(\alpha_m) \tag{72}$$

B.4 Labor Demand

Wages, W_{it} are firm-year specific and distributed log-normally: $\ln(W_t) \sim \mathcal{N}(0, \sigma_w^2)$. Then each firm chooses optimal labor according to:

$$L_{it} = \left(\alpha_\ell \frac{K_{it}^{\alpha_k}}{W_{it}} e^{\omega_{it}} \right)^{\frac{1}{1-\alpha_\ell}} \tag{73}$$

$$\text{In logs, } \ell_{it} = \frac{1}{1-\alpha_\ell} \left(\ln(\alpha_\ell) + \alpha_k k_{it} + \omega_{it} - \ln(W_{it}) \right) \tag{74}$$

B.5 Capital and Optimal Investment

Capital is accumulated as follows:

$$K_{it} = (1 - \delta) K_{it-1} + I_{t-1} \tag{75}$$

I set the depreciation rate at $\delta = 0.2$.

Investment is subject to convex adjustment costs $c(I_{it}) = \frac{b}{2} I_{it}^2$ with $b = 0.3$.

Optimal investment can be derived by setting up the profit maximization problem:[32]

$$\Pi_{it} = L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}} - W_{it} L_{it} - \frac{b}{2} I_{it}^2 \tag{76}$$

Here, I assume perfect competition and normalize the price of output to 1. The firm's value function is :

$$V(L_{it}, K_{it}, W_{it}, \omega_{it}) = \max_{L_{it}, K_{it}} L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}} - W_{it} L_{it} - \frac{b}{2} I_{it}^2 + \beta \, \mathbb{E}_{it} \, V(L_{it+1}, K_{it+1}, W_{it+1}, \omega_{it+1}) \tag{77}$$

$$\text{such that } K_{it+1} = (1-\delta) K_{it} + I_t \tag{78}$$

β is the discount factor and is fixed at 0.95. Optimal investment solves the Euler equation $\dfrac{\partial V}{\partial I} = 0$:

$$b I_{it} = \beta \, \mathbb{E}_{it} \, V_K(L_{it+1}, K_{it+1}, W_{it+1}, \omega_{it+1}) \tag{79}$$

The envelope condition yields:

$$V_K(L_{it}, K_{it}, W_{it}, \omega_{it}) = \alpha_k L_{it}^{\alpha_\ell} K_{it}^{\alpha_k - 1} e^{\omega_{it}} + \beta(1-\delta) \, \mathbb{E}_{it} \, V_K(L_{it+1}, K_{it+1}, W_{it+1}, \omega_{it+1}) \tag{80}$$

Substituting in (98) and (104):

$$V_K(L_{it}, K_{it}, W_{it}, \omega_{it}) = \alpha_k \alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}} K_{it}^{\frac{\alpha_k + \alpha_\ell - 1}{1-\alpha_\ell}} W_{it}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it}}{1-\alpha_\ell}} + b(1-\delta) I_{it} \tag{81}$$

Given a constant returns to scale technology ($\alpha_\ell + \alpha_k = 1$), the Euler equation becomes:

$$I_{it} = \frac{\beta \alpha_k}{b} \alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}} \, \mathbb{E}_{it} \left[W_{it+1}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it+1}}{1-\alpha_\ell}} \right] + \beta(1-\delta) \, \mathbb{E}_{it} \, I_{it+1} \tag{82}$$

$$\implies I_{it} = \frac{\beta \alpha_k}{b} \alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}} \sum_{\tau=0}^{\infty} \beta^\tau (1-\delta)^\tau \, \mathbb{E}_{it} \left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}} \right] \tag{83}$$

[32] This derivation follows Collard-Wexler and De Loecker (2016) and Van Biesebroeck (2007).

Since wages and productivity are drawn independently,

$$\mathbb{E}_{it}\left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right] = \mathbb{E}_{it}\left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}}\right]\mathbb{E}_t\left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right]$$

for all $\tau \geq 0$. Furthermore, $\ln(W_{it}) \sim \mathcal{N}(0,\sigma_w)^2 \implies \mathbb{E}_{it}\left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}}\right] = \exp\left(\frac{\alpha_\ell^2\sigma_w^2}{2(1-\alpha_\ell)^2}\right)$. The value of $\mathbb{E}_t\left[e^{\frac{\omega_{t+1+\tau}}{1-\alpha_\ell}}\right]$ depends on the productivity process:

$$\begin{aligned}
\omega_{t+1+\tau} =& \rho(I-\lambda G_{t+\tau+1})^{-1}\omega_{t+\tau} + (I-\lambda G_{t+\tau+1})^{-1}\varepsilon_{t+1+\tau} \\
=& \rho^2(I-\lambda G_{t+\tau+1})^{-1}(I-\lambda G_{t+\tau})^{-1}\omega_{it+\tau-1} + \rho(I-\lambda G_{t+\tau+1})^{-1}(I-\lambda W_{t+\tau})^{-1}\varepsilon_{t+\tau} \\
&+ (I-\lambda G_{t+\tau+1})^{-1}\varepsilon_{t+\tau+1} \\
\omega_{t+1+\tau} =& \rho^{\tau+1}\prod_{r=0}^{\tau}(I-\lambda G_{t+\tau+1-r})^{-1}\omega_t + \sum_{r=0}^{\tau}\rho^r\prod_{s=0}^{r}(I-\lambda G_{t+\tau+1-s})^{-1}\varepsilon_{t+\tau+1-r} \qquad (84)
\end{aligned}$$

$\mathbb{E}_t\left[e^{\frac{\omega_{t+1+\tau}}{1-\alpha_\ell}}\right]$ depends on the whether spillovers exist, and if they do, how firms form expectations about future links. When there are no spillovers $\lambda = 0$:

$$\begin{aligned}
\mathbb{E}_t\left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right] &= \mathbb{E}_t\left[\exp\left(\frac{\rho^{\tau+1}\omega_{it}}{1-\alpha_\ell} + \frac{1}{1-\alpha_\ell}\sum_{r=0}^{\tau}\rho^r\varepsilon_{t+\tau+1-r}\right)\right] \qquad (85) \\
&= \exp\left(\frac{\rho^{\tau+1}\omega_{it}}{1-\alpha_\ell}\right)\prod_{r=0}^{\tau}\mathbb{E}_{it}\left[\exp\left(\frac{\rho^r\varepsilon_{it+\tau+1-r}}{1-\alpha_\ell}\right)\right] \qquad (86) \\
&= \exp\left(\frac{\rho^{\tau+1}\omega_{it}}{1-\alpha_\ell}\right)\prod_{r=0}^{\tau}\exp\left(\frac{\rho^{2r}\sigma_\zeta^2}{2(1-\alpha_\ell)^2}\right) \qquad (87)
\end{aligned}$$

Let $\overline{G}$ represent the result of firms' beliefs about their future network. For example, if networks are non-stochastic or firms naively believe that $G_{t+\tau} = G_t \ \forall\tau > 0$, then we can set $\overline{G} = G_{t+1}$, which is deterministic given our previous assumption that

$G_{t+1} \in \mathcal{I}_t$:

$$\begin{aligned}
\mathbb{E}_t\left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right] &= \mathbb{E}_t\left[\exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t + \frac{1}{1-\alpha_\ell}\sum_{r=0}^{\tau}\rho^r(I-\lambda\overline{G})^{-(r+1)}\varepsilon_{t+\tau+1-r}\right)\right] \\
&= \exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t\right)\prod_{r=0}^{\tau}\mathbb{E}_{it}\left[\exp\left(\frac{\rho^r}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(r+1)}\varepsilon_{t+\tau+1-r}\right)\right] \\
&= \exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t\right)\prod_{r=0}^{\tau}\exp\left(\frac{\rho^{2r}\sigma_\zeta^2}{2(1-\alpha_\ell)^2}(I-\lambda\overline{G})^{-2(r+1)}\iota\right) \\
\mathbb{E}_{it}\left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right] &= \exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t\right)\prod_{r=0}^{\tau}\exp\left(\frac{\rho^{2r}\sigma_\zeta^2}{2(1-\alpha_\ell)^2(1-\lambda)^{2(r+1)}}\right)
\end{aligned} \tag{88}$$

Therefore, optimal investment choice reduces to a function of parameters and current productivity:

$$\begin{aligned}
I_t =& \frac{\beta\alpha_k}{b}\alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}}\exp\left(\frac{\alpha_\ell^2\sigma_w^2}{2(1-\alpha_\ell)^2}\right) \\
&\times\sum_{\tau=0}^{\infty}\beta^\tau(1-\delta)^\tau\exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t + \frac{\sigma_\zeta^2}{2(1-\alpha_\ell)^2(1-\lambda)^2}\sum_{r=0}^{\tau}\left(\frac{\rho}{1-\lambda}\right)^{2r}\right)
\end{aligned} \tag{89}$$

When there are no spillovers, this reduces to:

$$I_t = \frac{\beta\alpha_k}{b}\alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}}\exp\left(\frac{\alpha_\ell^2\sigma_w^2}{2(1-\alpha_\ell)^2}\right)\sum_{\tau=0}^{\infty}\beta^\tau(1-\delta)^\tau\exp\left(\frac{\rho^{\tau+1}\omega_t}{1-\alpha_\ell} + \frac{\sigma_\zeta^2\sum_{r=0}^{\tau}\rho^{2r}}{2(1-\alpha_\ell)^2}\right) \tag{90}$$

For alternative assumptions on the productivity process, such as a quadratic AR1 process, and endogenous network formation, it is not feasible to derive an closed-form solution as above. However, as long technology exhibits constant returns to scale, I approximate optimal investment as follows. Firstly, given $|\beta(1-\delta)| < 1$, then for some tolerance level close to zero, $\beta^\tau(1-\delta)^\tau <$ tolerance. Therefore, I can choose M sufficiently high such that $\sum_{\tau=0}^{M}\beta^\tau(1-\delta)^\tau\,\mathbb{E}_{it}\left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}}e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right]$ is a good approximation for $\sum_{\tau=0}^{\infty}\beta^\tau(1-\delta)^\tau\,\mathbb{E}_{it}\left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}}e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right]$. I set a tolerance level of e^{-4}, and given $\beta(1-\delta) = 0.95(1-0.2)$, then $M = 34$.

Next, at each time t, I draw 100 realizations of the sequence $\{\omega_{it+1+\tau}\}_{\tau=0}^{M}$ for each

firm i and approximate $\mathbb{E}_{it}\left[exp\left(\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}\right)\right] = \frac{1}{100}\sum_{s=0}^{100} exp\left(\frac{\omega_{it+1+\tau,s}}{1-\alpha_\ell}\right)$.

C Monte Carlo Setup

The Monte Carlo setup closely follows Collard-Wexler and De Loecker (2016), Van Biesebroeck (2007) and Ackerberg et al. (2015) with modifications for network generation and the inclusion of spillovers in the productivity process. I generate a balanced panel of 1000 firms over 10 time periods.

C.1 Production Function

I use a structural value-added production function that is Leontief in materials.

$$Y_{it} = \min\{L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}}, \alpha_m M_{it}\} e^{\varepsilon_{it}} \tag{91}$$

$$\implies Y_{it} = L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}+\varepsilon_{it}} = \alpha_m M_{it} e^{\varepsilon_{it}} \tag{92}$$

$$\text{In logs, } y_{it} = \alpha_\ell \ell_{it} + \alpha_k k_{it} + \omega_{it} + \varepsilon_{it} \tag{93}$$

where $\varepsilon_{it} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$. I set $\alpha_\ell = 0.6, \alpha_k = 0.4$ and $\sigma_\varepsilon^2 = 1$

C.2 Productivity Process and Network

Productivity evolves according to an AR1 process that allows for contemporaneous endogenous productivity spillovers. For ease of notation, I write the equation in vectorized form:

$$\omega_t = \beta_1 \iota + \rho \omega_{t-1} + \lambda G_t \omega_t + \zeta_t \tag{94}$$

where $\zeta_{it} \sim \mathcal{N}(0, \sigma_\zeta^2)$. I set $\sigma_\zeta^2 = 5$. I generate productivity using the reduced form of the above equation:

$$\omega_t = (\mathrm{I} - \lambda G_t)^{-1} (\beta_1 \iota + \rho \omega_{t-1} + \zeta_t) \tag{95}$$

G_t is the interaction matrix defined as in section 3.2 derived from the network. I generate exogenous networks using Erdős and Rényi (1960) graphs, also known as binomial graphs. Firms are edges are formed $A_{ijt} \overset{i.i.d.}{\sim} \text{Bern}(p)$.

C.3 Intermediate Input Demand

$$M_{it} = \frac{1}{\alpha_m} K_{it}^{\alpha_k} L_{it}^{\alpha_\ell} e^{\omega_{it}} \tag{96}$$

$$\text{In logs, } m_{it} = \alpha_k k_{it} + \alpha_\ell \ell_{it} + \omega_{it} - \ln(\alpha_m) \tag{97}$$

C.4 Labor Demand

Wages, W_{it} are firm-year specific and distributed log-normally: $\ln(W_t) \sim \mathcal{N}(0, \sigma_w^2)$. Then each firm chooses optimal labor according to:

$$L_{it} = \left(\alpha_\ell \frac{K_{it}^{\alpha_k}}{W_{it}} e^{\omega_{it}} \right)^{\frac{1}{1-\alpha_\ell}} \tag{98}$$

$$\text{In logs, } \ell_{it} = \frac{1}{1-\alpha_\ell} \left(\ln(\alpha_\ell) + \alpha_k k_{it} + \omega_{it} - \ln(W_{it}) \right) \tag{99}$$

C.5 Capital and Optimal Investment

Capital is accumulated as follows:

$$K_{it} = (1-\delta) K_{it-1} + I_{t-1} \tag{100}$$

I set the depreciation rate at $\delta = 0.2$.

Investment is subject to convex adjustment costs $c(I_{it}) = \frac{b}{2} I_{it}^2$ with $b = 0.3$. Optimal investment can be derived by setting up the profit maximization problem:[33]

$$\Pi_{it} = L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}} - W_{it} L_{it} - \frac{b}{2} I_{it}^2 \tag{101}$$

Here, I assume perfect competition and normalize the price of output to 1. The firm's value function is :

$$V(L_{it}, K_{it}, W_{it}, \omega_{it}) = \max_{L_{it}, K_{it}} L_{it}^{\alpha_\ell} K_{it}^{\alpha_k} e^{\omega_{it}} - W_{it} L_{it} - \frac{b}{2} I_{it}^2 + \beta \, \mathbb{E}_{it} \, V(L_{it+1}, K_{it+1}, W_{it+1}, \omega_{it+1}) \tag{102}$$

$$\text{such that } K_{it+1} = (1-\delta) K_{it} + I_t \tag{103}$$

[33] This derivation follows Collard-Wexler and De Loecker (2016) and Van Biesebroeck (2007).

β is the discount factor and is fixed at 0.95. Optimal investment solves the Euler equation $\dfrac{\partial V}{\partial I} = 0$:

$$bI_{it} = \beta\, \mathbb{E}_{it}\, V_K(L_{it+1}, K_{it+1}, W_{it+1}, \omega_{it+1}) \tag{104}$$

The envelope condition yields:

$$V_K(L_{it}, K_{it}, W_{it}, \omega_{it}) = \alpha_k L_{it}^{\alpha_\ell} K_{it}^{\alpha_k - 1} e^{\omega_{it}} + \beta(1-\delta)\, \mathbb{E}_{it}\, V_K(L_{it+1}, K_{it+1}, W_{it+1}, \omega_{it+1}) \tag{105}$$

Substituting in (98) and (104):

$$V_K(L_{it}, K_{it}, W_{it}, \omega_{it}) = \alpha_k \alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}} K_{it}^{\frac{\alpha_k+\alpha_\ell-1}{1-\alpha_\ell}} W_{it}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it}}{1-\alpha_\ell}} + b(1-\delta) I_{it} \tag{106}$$

Given a constant returns to scale technology ($\alpha_\ell + \alpha_k = 1$), the Euler equation becomes:

$$I_{it} = \frac{\beta\alpha_k}{b} \alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}}\, \mathbb{E}_{it} \left[W_{it+1}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it+1}}{1-\alpha_\ell}} \right] + \beta(1-\delta)\, \mathbb{E}_{it}\, I_{it+1} \tag{107}$$

$$\implies I_{it} = \frac{\beta\alpha_k}{b} \alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}} \sum_{\tau=0}^{\infty} \beta^\tau (1-\delta)^\tau\, \mathbb{E}_{it} \left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}} \right] \tag{108}$$

Since wages and productivity are drawn independently,

$$\mathbb{E}_{it} \left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}} \right] = \mathbb{E}_{it} \left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} \right] \mathbb{E}_t \left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}} \right]$$

for all $\tau \geq 0$. Furthermore, $\ln(W_{it}) \sim \mathcal{N}(0, \sigma_w)^2 \implies \mathbb{E}_{it} \left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}} \right] = \exp\left(\frac{\alpha_\ell^2 \sigma_w^2}{2(1-\alpha_\ell)^2} \right)$. The value of $\mathbb{E}_t \left[e^{\frac{\omega_{t+1+\tau}}{1-\alpha_\ell}} \right]$ depends on the productivity process:

$$\begin{aligned}
\omega_{t+1+\tau} =& \rho(I - \lambda G_{t+\tau+1})^{-1} \omega_{t+\tau} + (I - \lambda G_{t+\tau+1})^{-1} \varepsilon_{t+1+\tau} \\
=& \rho^2 (I - \lambda G_{t+\tau+1})^{-1} (I - \lambda G_{t+\tau})^{-1} \omega_{it+\tau-1} + \rho (I - \lambda G_{t+\tau+1})^{-1} (I - \lambda W_{t+\tau})^{-1} \varepsilon_{t+\tau} \\
&+ (I - \lambda G_{t+\tau+1})^{-1} \varepsilon_{t+\tau+1} \\
\omega_{t+1+\tau} =& \rho^{\tau+1} \prod_{r=0}^{\tau} (I - \lambda G_{t+\tau+1-r})^{-1} \omega_t + \sum_{r=0}^{\tau} \rho^r \prod_{s=0}^{r} (I - \lambda G_{t+\tau+1-s})^{-1} \varepsilon_{t+\tau+1-r}
\end{aligned} \tag{109}$$

$\mathbb{E}_t\left[e^{\frac{\omega_{t+1+\tau}}{1-\alpha_\ell}}\right]$ depends on the whether spillovers exist, and if they do, how firms form expectations about future links. When there are no spillovers $\lambda = 0$:

$$\mathbb{E}_t\left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right] = \mathbb{E}_t\left[\exp\left(\frac{\rho^{\tau+1}\omega_{it}}{1-\alpha_\ell} + \frac{1}{1-\alpha_\ell}\sum_{r=0}^{\tau}\rho^r\varepsilon_{t+\tau+1-r}\right)\right] \tag{110}$$

$$= \exp\left(\frac{\rho^{\tau+1}\omega_{it}}{1-\alpha_\ell}\right)\prod_{r=0}^{\tau}\mathbb{E}_{it}\left[\exp\left(\frac{\rho^r\varepsilon_{it+\tau+1-r}}{1-\alpha_\ell}\right)\right] \tag{111}$$

$$= \exp\left(\frac{\rho^{\tau+1}\omega_{it}}{1-\alpha_\ell}\right)\prod_{r=0}^{\tau}\exp\left(\frac{\rho^{2r}\sigma_\zeta^2}{2(1-\alpha_\ell)^2}\right) \tag{112}$$

Let $\overline{G}$ represent the result of firms' beliefs about their future network. For example, if networks are non-stochastic or firms naively believe that $G_{t+\tau} = G_t\ \forall\tau > 0$, then we can set $\overline{G} = G_{t+1}$, which is deterministic given our previous assumption that $G_{t+1} \in \mathcal{I}_t$:

$$\begin{aligned}
\mathbb{E}_t\left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right] &= \mathbb{E}_t\left[\exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t + \frac{1}{1-\alpha_\ell}\sum_{r=0}^{\tau}\rho^r(I-\lambda\overline{G})^{-(r+1)}\varepsilon_{t+\tau+1-r}\right)\right] \\
&= \exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t\right)\prod_{r=0}^{\tau}\mathbb{E}_{it}\left[\exp\left(\frac{\rho^r}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(r+1)}\varepsilon_{t+\tau+1-r}\right)\right] \\
&= \exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t\right)\prod_{r=0}^{\tau}\exp\left(\frac{\rho^{2r}\sigma_\zeta^2}{2(1-\alpha_\ell)^2}(I-\lambda\overline{G})^{-2(r+1)}\iota\right) \\
\mathbb{E}_{it}\left[e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right] &= \exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t\right)\prod_{r=0}^{\tau}\exp\left(\frac{\rho^{2r}\sigma_\zeta^2}{2(1-\alpha_\ell)^2(1-\lambda)^{2(r+1)}}\right)
\end{aligned} \tag{113}$$

Therefore, optimal investment choice reduces to a function of parameters and current productivity:

$$\begin{aligned}
I_t =& \frac{\beta\alpha_k}{b}\alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}}\exp\left(\frac{\alpha_\ell^2\sigma_w^2}{2(1-\alpha_\ell)^2}\right) \\
&\times\sum_{\tau=0}^{\infty}\beta^\tau(1-\delta)^\tau\exp\left(\frac{\rho^{\tau+1}}{1-\alpha_\ell}(I-\lambda\overline{G})^{-(\tau+1)}\omega_t + \frac{\sigma_\zeta^2}{2(1-\alpha_\ell)^2(1-\lambda)^2}\sum_{r=0}^{\tau}\left(\frac{\rho}{1-\lambda}\right)^{2r}\right)
\end{aligned} \tag{114}$$

When there are no spillovers, this reduces to:

$$I_t = \frac{\beta\alpha_k}{b}\alpha_\ell^{\frac{\alpha_\ell}{1-\alpha_\ell}} \exp\left(\frac{\alpha_\ell^2\sigma_w^2}{2(1-\alpha_\ell)^2}\right)\sum_{\tau=0}^{\infty}\beta^\tau(1-\delta)^\tau \exp\left(\frac{\rho^{\tau+1}\omega_t}{1-\alpha_\ell} + \frac{\sigma_\zeta^2\sum_{r=0}^{\tau}\rho^{2r}}{2(1-\alpha_\ell)^2}\right) \quad (115)$$

For alternative assumptions on the productivity process, such as a quadratic AR1 process, and endogenous network formation, it is not feasible to derive an closed-form solution as above. However, as long technology exhibits constant returns to scale, I approximate optimal investment as follows. Firstly, given $|\beta(1-\delta)| < 1$, then for some tolerance level close to zero, $\beta^\tau(1-\delta)^\tau <$ tolerance. Therefore, I can choose M sufficiently high such that $\sum_{\tau=0}^{M}\beta^\tau(1-\delta)^\tau\,\mathbb{E}_{it}\left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}}e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right]$ is a good approximation for $\sum_{\tau=0}^{\infty}\beta^\tau(1-\delta)^\tau\,\mathbb{E}_{it}\left[W_{it+1+\tau}^{\frac{-\alpha_\ell}{1-\alpha_\ell}}e^{\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}}\right]$. I set a tolerance level of e^{-4}, and given $\beta(1-\delta) = 0.95(1-0.2)$, then $M = 34$.

Next, at each time t, I draw 100 realizations of the sequence $\{\omega_{it+1+\tau}\}_{\tau=0}^{M}$ for each firm i and approximate $\mathbb{E}_{it}\left[exp\left(\frac{\omega_{it+1+\tau}}{1-\alpha_\ell}\right)\right] = \frac{1}{100}\sum_{s=0}^{100} exp\left(\frac{\omega_{it+1+\tau,s}}{1-\alpha_\ell}\right)$.

D Additional Monte Carlo Experiments

In this section, I consider how bias and precision change with the size of the endogenous network effect, and the persistence of productivity over time. The Monte Carlo setup is the same as in section **??**.

When I vary λ, ACF and ACF-N perform similarly, yielding comparable estimates of the input elasticities and spillover effects. Low values of λ are difficult to detect, while at very high values, there is a sharp decline in efficiency, with the decline greater under ACF. Under the linear process with negative spillovers, ACF-N appears to perform better than ACF as λ rises in magnitude.

Finally, variations in ρ_1 have striking effects on the estimation of λ because it determines the strength of $G_t\omega_{t-1}$ as an instrument for $G_t\omega_t$. Intuitively, if productivity is not persistent, then neighbors' lagged TFP is a weak instrument for the contemporaneous effect of neighbors's productivity, because the intertemporal correlation is not strong. ACF-N is not immune to this issue, and loses efficiency in its estimates of λ unless ρ_1 is sufficiently high. However the input elasticities are relatively well estimated by ACF-N, while ACF leads to biased estimates for high values of ρ_1: overestimating the capital coefficient when $\rho_1 = 0.8$ and underestimating it when $\rho_1 = 0.9$.

Table 13: Effect of λ on Bias and Precision (Quadratic AR1)

λ	Estimator		Elasticities		Productivity Process Coefficients			λ
			α_ℓ	α_k	β_1	ρ_1	ρ_2	
		True	0.6	0.4	0.5	0.8	-0.01	
0.01	ACF	Mean	0.603	0.353	-0.198	0.809	-0.01	0.
		Std. Dev.	(0.024)	(0.251)	(2.706)	(0.23)	(0.003)	(0.061)
	ACF-N	Mean	0.608	0.355	-0.038	0.811	-0.009	-0.02
		Std. Dev.	(0.037)	(0.234)	(2.394)	(0.212)	(0.027)	(0.337)
0.03	ACF	Mean	0.603	0.353	-0.182	0.809	-0.01	0.019
		Std. Dev.	(0.024)	(0.247)	(2.457)	(0.224)	(0.003)	(0.061)
	ACF-N	Mean	0.608	0.354	-0.03	0.812	-0.01	-0.009
		Std. Dev.	(0.035)	(0.23)	(2.218)	(0.206)	(0.003)	(0.686)
0.05	ACF	Mean	0.603	0.354	-0.16	0.808	-0.01	0.038
		Std. Dev.	(0.024)	(0.242)	(2.212)	(0.218)	(0.003)	(0.062)
	ACF-N	Mean	0.608	0.353	-0.027	0.813	-0.01	0.038
		Std. Dev.	(0.034)	(0.225)	(2.002)	(0.201)	(0.003)	(0.27)
0.07	ACF	Mean	0.604	0.354	-0.132	0.808	-0.01	0.057
		Std. Dev.	(0.024)	(0.237)	(1.978)	(0.212)	(0.003)	(0.062)
	ACF-N	Mean	0.607	0.354	0.013	0.814	-0.01	0.055
		Std. Dev.	(0.033)	(0.219)	(1.746)	(0.193)	(0.003)	(0.143)
0.09	ACF	Mean	0.604	0.357	-0.099	0.805	-0.01	0.076
		Std. Dev.	(0.024)	(0.23)	(1.759)	(0.204)	(0.003)	(0.063)
	ACF-N	Mean	0.607	0.355	0.044	0.814	-0.01	0.071
		Std. Dev.	(0.033)	(0.212)	(1.546)	(0.184)	(0.003)	(0.136)
0.1	ACF	Mean	0.604	0.36	-0.08	0.803	-0.01	0.086
		Std. Dev.	(0.024)	(0.226)	(1.65)	(0.2)	(0.003)	(0.063)
	ACF-N	Mean	0.607	0.356	0.059	0.814	-0.01	0.08
		Std. Dev.	(0.033)	(0.208)	(1.443)	(0.179)	(0.003)	(0.151)
0.3	ACF	Mean	0.606	0.643	0.054	0.414	-0.024	0.446
		Std. Dev.	(0.072)	(0.152)	(3.357)	(0.782)	(0.151)	(1.833)
	ACF-N	Mean	0.605	0.388	0.417	0.813	-0.01	0.299
		Std. Dev.	(0.032)	(0.084)	(0.399)	(0.065)	(0.004)	(0.062)
0.5	ACF	Mean	0.645	0.362	-5.659	0.723	-0.012	0.651
		Std. Dev.	(0.151)	(0.156)	(270.251)	(0.645)	(0.063)	(8.7)
	ACF-N	Mean	0.648	0.351	-3.503	0.77	-0.012	0.704
		Std. Dev.	(0.1)	(0.101)	(204.646)	(0.311)	(0.034)	(6.386)

0.7	ACF	Mean	0.682	0.318	-3.839	0.668	-0.012	1.148
		Std. Dev.	(0.19)	(0.19)	(217.5)	(4.683)	(0.616)	(11.871)
	ACF-N	Mean	0.685	0.314	4.003	0.592	-0.001	0.671
		Std. Dev.	(0.144)	(0.144)	(91.859)	(0.669)	(0.099)	(2.519)

Based on 1000 replications. Estimators are based on Ackerberg et al. (2015) with ACF denoting the standard procedure and ACF-N indicating the modified procedure to account for network effects. Networks are exogenous Erdos-Renyi (binomial) graphs with 0.05 density. The data-generating process for productivity is quadratic AR1 with endogenous network effects.

Table 14: Effect of ρ on Bias and Precision (Quadratic AR1)

ρ	Estimator		Elasticities		Productivity Process Coefficients			
			α_ℓ	α_k	β_1	ρ_1	ρ_2	λ
		True	0.6	0.4	0.5		-0.01	0.3
0.1	ACF	Mean	0.644	0.413	-271.942	0.485	-0.047	52.923
		Std. Dev.	(0.138)	(0.384)	(8697.007)	(12.757)	(1.038)	(1683.731)
	ACF-N	Mean	0.65	0.39	2.029	0.113	-0.009	0.151
		Std. Dev.	(0.1)	(0.365)	(90.035)	(0.514)	(0.041)	(10.864)
0.2	ACF	Mean	0.634	0.362	3.014	0.255	-0.003	-0.094
		Std. Dev.	(0.131)	(0.234)	(30.083)	(1.139)	(0.109)	(5.893)
	ACF-N	Mean	0.647	0.343	-0.796	0.205	-0.01	0.14
		Std. Dev.	(0.099)	(0.219)	(37.532)	(0.127)	(0.031)	(7.268)
0.3	ACF	Mean	0.628	0.369	1.427	0.286	-0.001	0.069
		Std. Dev.	(0.109)	(0.161)	(6.855)	(0.587)	(0.166)	(3.382)
	ACF-N	Mean	0.645	0.345	-0.132	0.312	-0.009	1.074
		Std. Dev.	(0.095)	(0.151)	(31.101)	(0.103)	(0.03)	(24.603)
0.4	ACF	Mean	0.617	0.381	1.412	0.314	0.028	-0.083
		Std. Dev.	(0.086)	(0.122)	(15.72)	(2.994)	(1.188)	(11.549)
	ACF-N	Mean	0.638	0.354	0.251	0.409	-0.01	0.45
		Std. Dev.	(0.084)	(0.118)	(21.734)	(0.086)	(0.028)	(8.122)
0.5	ACF	Mean	0.609	0.389	0.831	0.518	-0.013	0.268
		Std. Dev.	(0.061)	(0.094)	(1.139)	(0.193)	(0.104)	(0.935)
	ACF-N	Mean	0.628	0.364	1.104	0.512	-0.011	0.105
		Std. Dev.	(0.072)	(0.103)	(13.659)	(0.066)	(0.022)	(7.008)
0.6	ACF	Mean	0.606	0.398	0.758	0.609	-0.009	0.253
		Std. Dev.	(0.044)	(0.082)	(0.549)	(0.053)	(0.017)	(0.146)
	ACF-N	Mean	0.619	0.373	1.087	0.613	-0.01	0.186
		Std. Dev.	(0.057)	(0.09)	(6.734)	(0.064)	(0.016)	(1.727)
0.7	ACF	Mean	0.601	0.465	0.575	0.665	-0.012	0.29
		Std. Dev.	(0.043)	(0.086)	(0.38)	(0.081)	(0.014)	(0.1)
	ACF-N	Mean	0.608	0.385	0.642	0.714	-0.01	0.279
		Std. Dev.	(0.036)	(0.081)	(2.109)	(0.053)	(0.005)	(0.47)
0.8	ACF	Mean	0.606	0.643	0.054	0.414	-0.024	0.446
		Std. Dev.	(0.072)	(0.152)	(3.357)	(0.782)	(0.151)	(1.833)
	ACF-N	Mean	0.605	0.388	0.417	0.813	-0.01	0.299
		Std. Dev.	(0.032)	(0.084)	(0.399)	(0.065)	(0.004)	(0.062)
0.9	ACF	Mean	0.708	0.113	-5.964	0.65	-0.028	0.346
		Std. Dev.	(0.093)	(0.273)	(33.081)	(4.953)	(0.367)	(4.293)
	ACF-N	Mean	0.603	0.386	0.024	0.922	-0.01	0.296
		Std. Dev.	(0.028)	(0.09)	(1.165)	(0.122)	(0.002)	(0.033)

Based on 1000 replications. Estimators are based on Ackerberg et al. (2015) with ACF denoting the standard procedure and ACF-N indicating the modified procedure to account for network effects. Networks are exogenous Erdos-Renyi (binomial) graphs with 0.05 density. The data-generating process for productivity is quadratic AR1 with endogenous network effects.

E Variable Construction

- Sales: Net sales deflated by an industry deflator for GDP.
- Labor: Number of employees
- Capital: Total property, plant and equipment (gross) before depreciation. Following the method in İmrohoroğlu and Tüzel (2014), I deflate using the yearly implicit price deflator for fixed investment at the calculated age of capital. Capital age is computed as the ratio of accumulated depreciation to current depreciation, smoothed by taking a 3-year moving average. The year at which the deflator is applied is current year − average capital age. All years before 1929 are bottom-coded because that is the earliest year in the deflator data.
- Materials: Estimated as Cost of goods sold plus Selling, General and Administrative Expenses minus labor costs. Salaries and wage costs are missing for most firms, so I estimate labor costs by multiplying number of employees by 2-digit industry wages per full-time equivalent employee. Figure 18 shows that these estimates strongly correlate with wage costs that were reported in the data. Estimated materials are deflated by the 2-digit industry price indices for intermediate inputs.
- Value-added: Sales minus materials, deflated by industry price indices for value-added.
- Exports: International Sales as reported in the geographic segments information on annual reports. These figures are often reported by location of the final customer, but do not always differentiate between exports from the US and sales by multinational firms within foreign countries. However, to the extent that this contains some measure of exporting, a dummy for exporting based on positive values of this variable should have minimal measurement error.
- Industry: Industry classifications are based on those used in input-output tables from the Bureau of Economic Analysis (BEA). There are 65 industries from before 1997 and 71 industries from 1997 onwards. These roughly correspond to 3-digit NAICS and 2-digit SIC codes. Compustat's annual financials only reports the latest industry classification, therefore, I obtain historical NAICS

codes from the primary business segment. I also replace SIC codes for companies that are incorrectly coded as "99" (unclassifiable) from annual reports in the EDGAR database and business segment data. These are then converted to BEA industry codes using the concordances provided by the bureau. All deflators, price indices and input-output tables are based on these BEA industry codes. However, in regressions I combine industries with too few observations. These include: transit and ground transportation with general transportation and warehousing, and other transportation and support activities; Funds, trusts and other financial vehicles combined with securities, commodity contracts and investments; Legal services with miscellaneous professional services; Ambulatory health, hospitals, nursing and residential care with social assistance. This results in 41 industry groups.



Figure 18: Estimated and Reported Labor Expenses

This figure shows the correlation between labor expenses reported in *Compustat* and labor costs estimated using industry wage expenditure.

F Additional Results and Robustness Checks

F.1 Value-Added Estimates

Table 15: Productivity Spillovers by Relationship Dynamics
(Value-Added)

		Dependent Variable: $\ln TFP_t$			
Period	Estimator	Continuing Customers' $\ln TFP_t$	New Customers' $\ln TFP_t$	Continuing Suppliers' $\ln TFP_t$	New Suppliers' $\ln TFP_t$
1977-1986	ACF	0.0009	0.0023	0.002	0.0005
		(0.0014)	(0.0014)	(0.0011)	(0.001)
	ACF-N	0.0008	0.0022	0.0021	0.0007
		(0.0014)	(0.0014)	(0.0011)	(0.001)
	ACF-ND	0.0008	0.001	0.003	0.0003
		(0.0016)	(0.0015)	(0.0013)	(0.0011)
1987-1996	ACF	0.0001	0.0003	-0.0006	0.001
		(0.0009)	(0.001)	(0.0007)	(0.0006)
	ACF-N	0.0002	0.0003	-0.0011	0.0005
		(0.0009)	(0.001)	(0.0007)	(0.0006)
	ACF-ND	0.0004	0.0004	-0.0012	0.0006
		(0.001)	(0.001)	(0.0007)	(0.0007)
1997-2006	ACF	0.0008	0.0004	0.0003	0.0014
		(0.0005)	(0.0005)	(0.0004)	(0.0004)
	ACF-N	0.0008	0.0004	0.0006	0.0017
		(0.0005)	(0.0005)	(0.0004)	(0.0004)
	ACF-ND	0.0005	0.0004	0.0006	0.002
		(0.0005)	(0.0004)	(0.0005)	(0.0004)
2007-2016	ACF	0.0012	0.0003	0.0011	0.0003
		(0.0006)	(0.0004)	(0.0004)	(0.0003)
	ACF-N	0.0011	0.0003	0.0013	0.0004
		(0.0006)	(0.0004)	(0.0004)	(0.0003)
	ACF-ND	0.0012	0.0003	0.0011	0.0002
		(0.0007)	(0.0004)	(0.0004)	(0.0003)
All	ACF	0.0007	0.0008	0.0006	0.0009
		(0.0003)	(0.0003)	(0.0003)	(0.0002)
	ACF-N	0.0007	0.0008	0.0008	0.001
		(0.0003)	(0.0003)	(0.0003)	(0.0002)
	ACF-ND	0.0007	0.0008	0.0007	0.0011
		(0.0004)	(0.0003)	(0.0003)	(0.0002)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a value-added production function (in logs) estimated with the standard Ackerberg et al. (2015) procedure (ACF), or with modifications to accommodate network effects (ACF-N) and network differencing (ACF-ND). Network effects for ACF are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 16: Productivity Spillovers by Sector (Value-Added)

	Dependent Variable: $\ln TFP_t$											
	Firm's Sector											
Partners' Sector	Mining	Utilities	Constr	Durables	Non-Durables	Electronics	Wholesale	Retail	Trans & WH	Info	FIRE	Services
Mining	0.0037	-0.0004	0.009	0.0027	0.0014	-0.0057	-0.0011	-0.0026	0.0121	0.0135	-0.001	-0.0035
	(0.0018)	(0.0011)	(0.0092)	(0.0013)	(0.0009)	(0.0081)	(0.0034)	(0.0025)	(0.0055)	(0.0068)	(0.0036)	(0.0025)
Utilities	0.0029	0.0025	0.0034	-0.0001	0.0022	0.0025	0.0052	-0.0023	0.0024	-0.0013	-0.003	0.0033
	(0.0018)	(0.001)	(0.004)	(0.001)	(0.0011)	(0.0021)	(0.0033)	(0.0028)	(0.0021)	(0.0054)	(0.0107)	(0.0034)
Construction	0.0151	0.0014	0.009	0.0013	-0.0014	-0.0018	-0.0107	-0.0057	0.0047	0.0004	0.001	0.0096
	(0.0172)	(0.0013)	(0.0049)	(0.0023)	(0.0009)	(0.0032)	(0.0068)	(0.0037)	(0.0091)	(0.001)	(0.0033)	(0.0107)
Durables Mfg	0.0008	-0.0007	0.0171	-0.0002	-0.0019	-0.0017	0.0013	-0.0014	-0.0002	-0.0011	0.0027	0.003
	(0.0018)	(0.0008)	(0.0084)	(0.0006)	(0.0006)	(0.0006)	(0.0012)	(0.0007)	(0.0013)	(0.0013)	(0.0035)	(0.0014)
Non-Durables Mfg	0.0023	0.0007	0.0048	-0.0011	-0.0011	-0.0006	0.003	-0.0043	0.0035	-0.0012	-0.0073	-0.0059
	(0.0019)	(0.0011)	(0.0061)	(0.0006)	(0.0005)	(0.0011)	(0.0012)	(0.0007)	(0.0019)	(0.0016)	(0.0035)	(0.0017)
Electronics Mfg	-0.0087	-0.0011	-0.008	0.0004	-0.0014	0.0012	0.0042	-0.0013	0.0008	-0.0006	0.0049	-0.0
	(0.0081)	(0.0014)	(0.0084)	(0.0006)	(0.0012)	(0.0006)	(0.0011)	(0.0013)	(0.0013)	(0.0008)	(0.004)	(0.0014)
Wholesale	0.0007	-0.0005	0.0214	0.0007	0.001	0.0024	0.0061	-0.002	-0.0022	-0.0004	-0.0068	-0.0012
	(0.0042)	(0.0013)	(0.0216)	(0.0007)	(0.0006)	(0.0008)	(0.0025)	(0.0009)	(0.0026)	(0.0012)	(0.0058)	(0.0015)
Retail	-0.0052	-0.0082	0.0023	0.0018	0.0006	0.005	0.0033	0.0001	-0.0012	0.0044	0.0086	0.0002
	(0.0039)	(0.0032)	(0.0169)	(0.0007)	(0.0006)	(0.0018)	(0.0016)	(0.0012)	(0.0013)	(0.0022)	(0.0032)	(0.0022)
Transport and Warehousing	0.0031	-0.0003	0.013	0.0006	0.0005	-0.0016	0.0162	0.0025	0.0004	-0.0	0.0161	-0.0034
	(0.0029)	(0.0008)	(0.0084)	(0.0013)	(0.0006)	(0.0029)	(0.0059)	(0.0019)	(0.0016)	(0.0019)	(0.0044)	(0.0024)
Information	0.0071	0.0002	0.0036	-0.0026	-0.0017	0.0011	0.0055	-0.0003	-0.0042	0.002	0.0029	-0.0022
	(0.0052)	(0.0013)	(0.0064)	(0.0012)	(0.0011)	(0.0007)	(0.0021)	(0.0014)	(0.0014)	(0.0009)	(0.0019)	(0.0015)
Finance, Insur & Real Estate	0.002	-0.0049	-0.0054	-0.0007	-0.0008	0.0031	0.0038	-0.0022	0.0027	-0.0013	-0.0008	0.0009
	(0.0034)	(0.0045)	(0.003)	(0.0012)	(0.0006)	(0.0009)	(0.0066)	(0.001)	(0.0012)	(0.0012)	(0.0017)	(0.0015)
Services	-0.0006	-0.0004	0.0055	0.0004	-0.001	0.0003	-0.0004	0.0003	-0.0014	0.0004	0.0011	0.0007
	(0.0027)	(0.001)	(0.015)	(0.0008)	(0.0006)	(0.0007)	(0.002)	(0.0016)	(0.0015)	(0.0012)	(0.0017)	(0.0015)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Ackerberg et al. (2015) procedure (ACF), or with modifications to accommodate network effects and network differencing (ACF-ND). Network effects for ACF are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Sectors are determined according to the BEA industry classification. Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 17: Productivity Spillovers by Firm Size & Relationship Direction (Value-Added)

Partner Size	Relationship	Firm Size	Dependent Variable: $\ln TFP_t$ 1977-1986	1987-1996	1997-2006	2007-2016	All
Large	Customers	Large	0.0002	0.0005	0.0002	0.0007	0.0008
			(0.0015)	(0.0007)	(0.0005)	(0.0004)	(0.0003)
		Small	0.0066	0.0007	-0.0021	-0.0001	0.002
			(0.0019)	(0.001)	(0.0007)	(0.0006)	(0.0004)
	Suppliers	Large	0.0002	-0.0009	0.0012	0.0007	0.0004
			(0.0013)	(0.0006)	(0.0004)	(0.0003)	(0.0002)
		Small	0.0339	0.0006	-0.0012	-0.0038	0.0028
			(0.018)	(0.005)	(0.0046)	(0.0031)	(0.0027)
Small	Customers	Large	-0.0033	-0.0046	-0.0088	-0.0043	-0.0054
			(0.0047)	(0.0048)	(0.004)	(0.0022)	(0.0023)
		Small	0.0125	-0.0059	-0.0021	0.0022	-0.0003
			(0.0071)	(0.0062)	(0.0029)	(0.0059)	(0.0028)
	Suppliers	Large	0.0013	-0.0001	0.0017	0.0005	0.0007
			(0.0011)	(0.0007)	(0.0005)	(0.0003)	(0.0002)
		Small	0.0218	-0.0004	0.0045	-0.0023	0.0039
			(0.0088)	(0.0046)	(0.0041)	(0.0046)	(0.0025)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a value-added production function (in logs) estimated with the standard Ackerberg et al. (2015) procedure (ACF), or with modifications to accommodate network effects (ACF-N) and network differencing (ACF-ND). Network effects for ACF are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Large firms are businesses with 500 or more employees.Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 18: Productivity Spillovers by Varying Firm Size Cutoffs
(Value-Added)

Dependent Variable: $\ln TFP_t$						
			Firm's Sector			
Partner Size	Relationship	Firm Size	500	1000	5000	Median
Large	Customers	Large	0.0008	0.0008	-0.0	0.0002
			(0.0003)	(0.0003)	(0.0003)	(0.0003)
		Small	0.002	0.003	0.0014	0.0022
			(0.0004)	(0.0004)	(0.0004)	(0.0004)
	Suppliers	Large	0.0004	0.0002	-0.0006	-0.0
			(0.0002)	(0.0002)	(0.0002)	(0.0002)
		Small	0.0028	0.0049	0.0037	0.0048
			(0.0027)	(0.0015)	(0.001)	(0.0006)
Small	Customers	Large	-0.0054	-0.002	-0.0009	-0.0003
			(0.0023)	(0.0015)	(0.0009)	(0.0005)
		Small	-0.0003	-0.0014	0.0004	0.0019
			(0.0028)	(0.0018)	(0.0006)	(0.0007)
	Suppliers	Large	0.0007	0.0006	-0.0001	0.0
			(0.0002)	(0.0002)	(0.0003)	(0.0003)
		Small	0.0039	0.0038	0.0039	0.0032
			(0.0025)	(0.0015)	(0.0006)	(0.0006)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a value-added production function (in logs) estimated with the standard Ackerberg et al. (2015) procedure (ACF), or with modifications to accommodate network effects (ACF-N) and network differencing (ACF-ND). Network effects for ACF are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Large firms are defined by having at least as many employees as the cutoffs indicated above. The median cutoff is determined by industry and year. Standard errors are in parentheses. All specifications include industry and year fixed effects.

F.2 Unweighted Estimates

Table 19: Endogenous Productivity Spillovers (Gross Output, Unweighted)

Dependent Variable: $\ln TFP_t$			
Period	Estimator	$\ln TFP_{t-1}$	Neighbors' $\ln TFP_t$
1977-1986	GNR	0.8403	0.0068
		(0.0205)	(0.0049)
	GNR-N	0.8391	0.0074
		(0.0207)	(0.0049)
	GNR-ND	0.8248	-0.0001
		(0.0228)	(0.0058)
1987-1996	GNR	0.8314	-0.0065
		(0.0244)	(0.0039)
	GNR-N	0.8312	-0.0063
		(0.0244)	(0.0039)
	GNR-ND	0.8232	-0.0093
		(0.0271)	(0.0039)
1997-2006	GNR	0.8583	0.0001
		(0.013)	(0.0043)
	GNR-N	0.8588	0.0003
		(0.0132)	(0.0043)
	GNR-ND	0.8584	0.0002
		(0.0145)	(0.0055)
2007-2016	GNR	0.8964	0.0115
		(0.0214)	(0.0048)
	GNR-N	0.8964	0.0113
		(0.0214)	(0.005)
	GNR-ND	0.889	0.0096
		(0.0229)	(0.0047)
All	GNR	0.9038	0.0076
		(0.0066)	(0.0023)
	GNR-N	0.9027	0.0086
		(0.0067)	(0.0024)
	GNR-ND	0.8998	0.0069
		(0.0073)	(0.0025)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure in Lee (2003); Bramoullé et al. (2009). Interaction matrices for network effects are unweighted. Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 20: Productivity Spillovers by Relationship Direction (Gross Output, Unweighted)

Dependent Variable: $\ln TFP_t$			
Period	Estimator	Customers' $\ln TFP_t$	Suppliers' $\ln TFP_t$
1977-1986	GNR	0.0036	0.0167
		(0.004)	(0.005)
	GNR-N	0.0022	0.017
		(0.0033)	(0.0045)
	GNR-ND	-0.0025	0.0127
		(0.0036)	(0.004)
1987-1996	GNR	0.0063	-0.008
		(0.004)	(0.0036)
	GNR-N	0.0079	-0.0122
		(0.0039)	(0.0035)
	GNR-ND	-0.0056	-0.012
		(0.0044)	(0.0079)
1997-2006	GNR	0.0013	0.0023
		(0.0005)	(0.0005)
	GNR-N	0.001	0.0033
		(0.0005)	(0.0005)
	GNR-ND	0.0006	0.0041
		(0.0006)	(0.0006)
2007-2016	GNR	0.0006	0.0016
		(0.0004)	(0.0006)
	GNR-N	0.0004	0.0024
		(0.0004)	(0.0008)
	GNR-ND	0.0002	0.0025
		(0.0004)	(0.0008)
All	GNR	0.0025	0.0053
		(0.0008)	(0.0009)
	GNR-N	0.0024	0.0079
		(0.0008)	(0.0011)
	GNR-ND	0.0015	0.008
		(0.0008)	(0.001)

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Table 21: Productivity Spillovers by Relationship Dynamics (Gross Output, Unweighted)

		Dependent Variable: $\ln TFP_t$			
Period	Estimator	Continuing Customers' $\ln TFP_t$	New Customers' $\ln TFP_t$	Continuing Suppliers' $\ln TFP_t$	New Suppliers' $\ln TFP_t$
1977-1986	GNR	0.0037	0.001	0.0149	0.0092
		(0.0034)	(0.0039)	(0.0038)	(0.0039)
	GNR-N	0.0013	0.0004	0.0138	0.0095
		(0.0024)	(0.0026)	(0.0029)	(0.0028)
	GNR-ND	0.0001	-0.0028	0.0157	0.0088
		(0.0031)	(0.0031)	(0.0032)	(0.0035)
1987-1996	GNR	0.0077	0.0033	-0.003	-0.0121
		(0.0035)	(0.0039)	(0.0038)	(0.0037)
	GNR-N	-0.0036	0.0165	-0.0041	-0.0295
		(0.0048)	(0.0068)	(0.0081)	(0.0077)
	GNR-ND	-0.0116	0.0101	0.0006	-0.0214
		(0.0046)	(0.0065)	(0.009)	(0.0088)
1997-2006	GNR	-0.0007	0.0005	0.0003	0.0013
		(0.0004)	(0.0004)	(0.0003)	(0.0003)
	GNR-N	-0.0009	0.0003	0.0015	0.0026
		(0.0004)	(0.0004)	(0.0004)	(0.0004)
	GNR-ND	-0.0011	0.0002	0.002	0.0031
		(0.0005)	(0.0005)	(0.0005)	(0.0004)
2007-2016	GNR	0.0005	0.0005	0.0015	0.0013
		(0.0004)	(0.0004)	(0.0005)	(0.0004)
	GNR-N	0.0004	0.0004	0.0025	0.0022
		(0.0004)	(0.0004)	(0.0007)	(0.0005)
	GNR-ND	0.0001	0.0003	0.0025	0.0022
		(0.0004)	(0.0004)	(0.0008)	(0.0006)
All	GNR	0.0008	0.0017	0.0029	0.0038
		(0.0006)	(0.0006)	(0.0006)	(0.0006)
	GNR-N	0.0009	0.0017	0.0066	0.0076
		(0.0007)	(0.0007)	(0.0009)	(0.0008)
	GNR-ND	0.0005	0.0012	0.007	0.0081
		(0.0007)	(0.0008)	(0.001)	(0.0009)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Interaction matrices for network effects are unweighted. Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 22: Productivity Spillovers by Sector (Gross Output, Unweighted)

	Dependent Variable: $\ln TFP_t$ Firm's Sector											
Partners' Sector	Mining	Utilities	Constr	Durables	Non-Durables	Electronics	Wholesale	Retail	Trans & WH	Info	FIRE	Services
Mining	-0.0082	-0.0093	-0.0062	0.0017	0.0071	-0.0155	0.0165	0.0088	0.0098	0.0343	0.0042	0.0027
	(0.0062)	(0.0037)	(0.0317)	(0.0048)	(0.0026)	(0.0312)	(0.01)	(0.0077)	(0.0088)	(0.0107)	(0.0082)	(0.0077)
Utilities	-0.0031	-0.0006	0.006	0.0029	0.0097	0.0127	0.0041	-0.0027	0.0018	-0.0023	-0.0035	0.01
	(0.0057)	(0.0027)	(0.0102)	(0.0026)	(0.002)	(0.0079)	(0.008)	(0.0081)	(0.0063)	(0.0121)	(0.0173)	(0.0058)
Construction	-0.0189	0.0012	0.0144	0.0052	-0.0057	-0.0076	0.0109	-0.0072	0.0164	0.0041	-0.0002	0.0276
	(0.0303)	(0.004)	(0.0112)	(0.0057)	(0.0035)	(0.0128)	(0.0073)	(0.0087)	(0.0222)	(0.0034)	(0.0059)	(0.0213)
Durables Mfg	0.0074	0.0002	0.0255	0.0024	0.0012	-0.0036	0.0076	-0.0016	0.0028	0.0037	0.0049	0.003
	(0.0063)	(0.0029)	(0.0223)	(0.0021)	(0.0016)	(0.0023)	(0.0026)	(0.0016)	(0.0034)	(0.0033)	(0.0061)	(0.0033)
Non-Durables Mfg	-0.0059	-0.0014	-0.0028	0.0	-0.0009	-0.0065	0.0033	-0.006	-0.0063	-0.0025	-0.0169	-0.0098
	(0.005)	(0.0027)	(0.0129)	(0.0017)	(0.0013)	(0.0046)	(0.0026)	(0.0015)	(0.0037)	(0.0043)	(0.0082)	(0.0037)
Electronics Mfg	-0.0413	-0.015	-0.0262	-0.0001	-0.004	0.0231	0.0126	0.0021	0.0076	0.0005	0.0203	0.0002
	(0.0478)	(0.0053)	(0.034)	(0.0044)	(0.0036)	(0.0028)	(0.0029)	(0.0031)	(0.0055)	(0.0029)	(0.0089)	(0.0041)
Wholesale	-0.005	0.0122	0.0115	0.0021	0.0031	0.016	0.0096	0.0016	0.0032	0.0021	-0.0125	0.0026
	(0.0114)	(0.0073)	(0.0157)	(0.002)	(0.0012)	(0.002)	(0.0037)	(0.0013)	(0.0101)	(0.0023)	(0.0114)	(0.0038)
Retail	0.0109	-0.0035	-0.0072	0.0071	0.0029	0.0187	0.0101	0.0012	-0.0012	0.0086	0.0175	0.0016
	(0.0114)	(0.01)	(0.0154)	(0.0027)	(0.0013)	(0.0035)	(0.0018)	(0.0025)	(0.0038)	(0.004)	(0.0045)	(0.0043)
Transport and Warehousing	0.0075	0.0111	0.0536	0.0035	0.0031	0.0063	0.0074	0.0021	-0.003	0.0007	-0.0084	0.0
	(0.009)	(0.0032)	(0.0183)	(0.004)	(0.0019)	(0.0057)	(0.0105)	(0.0038)	(0.004)	(0.0079)	(0.007)	(0.01)
Information	0.0235	-0.0023	0.0083	-0.0034	-0.001	0.0145	0.0062	-0.001	-0.0174	0.004	0.0064	-0.0071
	(0.0114)	(0.0066)	(0.0165)	(0.0038)	(0.0027)	(0.0024)	(0.0047)	(0.0025)	(0.0046)	(0.0027)	(0.0046)	(0.0038)
Finance, Insur & Real Estate	0.0003	0.0048	-0.013	-0.0	-0.0044	0.0129	-0.0024	0.0017	0.0043	0.004	-0.0019	0.0047
	(0.0108)	(0.0137)	(0.0124)	(0.0043)	(0.0024)	(0.0033)	(0.0144)	(0.0019)	(0.0042)	(0.003)	(0.0042)	(0.0034)
Services	0.0032	-0.0086	0.0082	0.0084	0.004	0.0021	0.008	0.004	0.0023	0.005	0.0088	0.0024
	(0.0113)	(0.0035)	(0.0245)	(0.0022)	(0.0022)	(0.0027)	(0.0069)	(0.0025)	(0.0048)	(0.003)	(0.0047)	(0.0036)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Sectors are determined according to the BEA industry classification. Interaction matrices for network effects are unweighted. Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 23: Productivity Spillovers by Firm Size & Relationship Direction (Gross Output, Unweighted)

Partner Size	Relationship	Firm Size	Dependent Variable: $\ln TFP_t$ 1977-1986	1987-1996	1997-2006	2007-2016	All
Large	Customers	Large	0.001	0.0028	-0.0001	0.0003	0.0019
			(0.0034)	(0.0073)	(0.0005)	(0.0004)	(0.0008)
		Small	-0.0039	-0.0019	0.0006	-0.0004	0.0019
			(0.0042)	(0.0056)	(0.0006)	(0.0006)	(0.001)
	Suppliers	Large	0.0151	-0.0252	0.0025	0.0026	0.0081
			(0.0036)	(0.0108)	(0.0004)	(0.0007)	(0.001)
		Small	0.0569	-0.0686	0.0047	-0.005	0.0139
			(0.0304)	(0.0216)	(0.0025)	(0.0025)	(0.0076)
Small	Customers	Large	-0.0134	-0.0778	-0.0063	-0.0059	-0.009
			(0.0117)	(0.0468)	(0.0028)	(0.0064)	(0.0075)
		Small	0.0002	-0.051	0.0	-0.0008	-0.0044
			(0.0143)	(0.0392)	(0.0022)	(0.0027)	(0.0052)
	Suppliers	Large	0.0138	-0.0125	0.0023	0.0022	0.008
			(0.0039)	(0.0058)	(0.0005)	(0.0006)	(0.0009)
		Small	0.0581	-0.0154	0.0033	0.0	0.0074
			(0.0272)	(0.0137)	(0.002)	(0.003)	(0.0052)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for ACF are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Large firms are businesses with 500 or more employees.Interaction matrices for network effects are unweighted. Standard errors are in parentheses. All specifications include industry and year fixed effects.

Table 24: Productivity Spillovers by Varying Firm Size Cutoffs (Gross Output, Unweighted)

Dependent Variable: $\ln TFP_t$						
			Firm's Sector			
Partner Size	Relationship	Firm Size	500	1000	5000	Median
Large	Customers	Large	0.0019	0.0016	0.0014	0.001
			(0.0008)	(0.0008)	(0.0008)	(0.0008)
		Small	0.0019	0.0033	0.0017	0.0014
			(0.001)	(0.0011)	(0.001)	(0.001)
	Suppliers	Large	0.0081	0.0078	0.006	0.0087
			(0.001)	(0.001)	(0.0008)	(0.001)
		Small	0.0139	0.0093	0.0084	0.0091
			(0.0076)	(0.0035)	(0.0027)	(0.0016)
Small	Customers	Large	-0.009	-0.001	-0.0039	0.0018
			(0.0075)	(0.006)	(0.0038)	(0.0015)
		Small	-0.0044	-0.0051	0.0007	0.0026
			(0.0052)	(0.0036)	(0.0012)	(0.0012)
	Suppliers	Large	0.008	0.0086	0.0079	0.0073
			(0.0009)	(0.001)	(0.0012)	(0.0011)
		Small	0.0074	0.0071	0.009	0.0087
			(0.0052)	(0.0032)	(0.0013)	(0.0013)

This table reports coefficients of a linear productivity evolution process with endogenous network effects estimated on US firms in Compustat. Each TFP measure is from a gross output production function (in logs) estimated with the standard Gandhi et al. (2020) procedure (GNR), or with modifications to accommodate network effects (GNR-N) and network differencing (GNR-ND). Network effects for GNR are estimated using the generalized 2SLS procedure for heterogenous peer effects in Dieye and Fortin (2017); Patacchini et al. (2017). Large firms are defined by having at least as many employees as the cutoffs indicated above. The median cutoff is determined by industry and year. Interaction matrices for network effects are unweighted. Standard errors are in parentheses. All specifications include industry and year fixed effects.